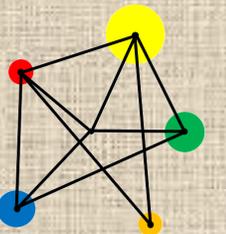# Compound Percentages

Review basic percentages

Calculate repeated percentage change

Vocabulary

Compound
A quantity made up of more than one other quantity.

Interest
A charge made for borrowing money. It is usually charged as a percentage of the borrowed amount.

Review of Percentages: Non Calculator

Find these percentages.

1. 50% of 120

2. 100% of 37

3. 25% of 36

4. 75% of 44

5. 10% of 82

6. 1% of 5000

Find these percentages.

1. 5% of 120

2. 15% of 40

3. 35% of 10

4. 11% of 800

5. 9% of 60

Find these percentages.

1. 17.5% of 800

2. 105% of 4000

3. $12\frac{4}{5}$ % of 10

4. 300% of 90

Review of Percentages: Non Calculator

Find these percentages.

1. 50% of 120 **= 60**

2. 100% of 37 **= 37**

3. 25% of 36 **= 9**

4. 75% of 44 **= 33**

5. 10% of 82 **= 8.2**

6. 1% of 5000 **= 50**

Find these percentages.

1. 5% of 120 **= 6**

2. 15% of 40 **= 6**

3. 35% of 10 **= 3.5**

4. 11% of 800 **= 88**

5. 9% of 60 **= 5.4**

Find these percentages.

1. 17.5% of 800 **= 140**

2. 105% of 4000 **= 4200**

3. $12\frac{4}{5}$ % of 10 **= 1.28**

4. 300% of 90 **= 270**

Percentage Multipliers

What single number could you multiply by to find the following percentages?

a) 89%

b) 20%

c) 100%

d) 7%

e) 1.4%

f) 221%

Solutions

What single number could you multiply by to find the following percentages?

a) 89%   **0.89**

b) 20%   **0.2**

c) 100%   **1**

d) 7%   **0.07**

e) 1.4%   **0.014**

f) 221%   **2.21**

Percentage Multipliers

What multiplier would you use to:

1. Find 50%

2. Find 25%

3. Find 16%

4. Find 10%

5. Find 4%

6. Find 100%

What multiplier would you use to:

1. Find 105%

2. Increase by 7%

3. Decrease by 15%

4. Increase by 100%

5. Reduce by 1%

What multiplier would you use to:

1. Increase by $17\frac{1}{2}$ %

2. Decrease by one fifth

3. Find $\frac{7}{8}$

4. Halve and halve again.

Percentage Multipliers

What multiplier would you use to:

1. Find 50% **0.5**

2. Find 25% **0.25**

3. Find 16% **0.16**

4. Find 10% **0.1**

5. Find 4% **0.04**

6. Find 100% **1**

What multiplier would you use to:

1. Find 105% **1.05**

2. Increase by 7% **1.07**

3. Decrease by 15% **0.85**

4. Increase by 100% **2**

5. Reduce by 1% **0.99**

What multiplier would you use to:

1. Increase by $17\frac{1}{2}\%$ **1.175**

2. Decrease by one fifth **0.8**

3. Find $\frac{7}{8}$ **0.875**

4. Halve and halve again. **0.25**

Example

A tree is 10m tall.

Each year its height increases by 2%.

How tall is the tree after:

- 1 year
- 2 years
- 4 years
- 10 years

Solution

A tree is 10m tall.

Each year its height increases by 2%

After 1 year   10 x 1.02 = 10.2m

After 2 years   10.2 x 1.02 = 10.404m

After 3 years   10.404 x 1.02 = 10.61208m

After 4 years   10.61208 x 1.02 = 10.8243216m

Example

A tree is 10m tall.

Each year its height increases by 2%.

How tall is the tree after 10 years?

Example

A tree is 10m tall.

Each year its height increases by 2%.

How tall is the tree after 10 years?

. . . .

10 x 1.02 x 1.02 x 1.02 x 1.02 x 1.02 x 1.02 x 1.02 x 1.02 x 1.02 x 1.02

$= 10 \times 1.02^{10}$

= 12.19 m

Key Fact

If a quantity repeatedly changes by the same percentage 'n' times, then we can evaluate its final value as follows.

$$Final\ Value\ =\ Initial\ Value\ \times (Multiplier)^n$$

Example

Amelie invests £400 in a bank account for three years.

She receives compound interest of 2% per annum.

How much will she have in her account at the end of 3 years?

Solution

Amelie invests £400 in a bank account for three years.

She receives compound interest of 2% per annum.

How much will she have in her account at the end of 3 years?

Amelie will have 400 x 1.02^3

= £424.48

Exercise

Complete the table

Initial value of investment	Interest Rate (%)	Number of Years	Final Value of Investment
£1000	5	3	
£1900	1.5	4	
£1 000 000	1	100	
$1800	10		$3188.81
£200 000		5	£220 816.16
	6	3	£119 101.60

Solutions

Complete the table

Initial value of investment	Interest Rate (%)	Number of Years	Final Value of Investment
£1000	5	3	**£1157.63**
£1900	1.5	4	**£2016.59**
£1 000 000	1	100	**£2704813.83**
$1800	10	**6**	$3188.81
£200 000	**2**	5	£220 816.16
£100 000	6	3	£119 101.60

Question 1

Josh invests £800 in a bank account for two years.

He receives compound interest of 3%

How much does he have in his account at the end of two years?

Question 2

The population of a country was 10 million at the start of 2010.

It has been increasing at a rate of 5.4% per year.

What was the population at the start of 2022?

Question 3

The price of a subscription to website A is £100 and it increases by 8% each year.

The price of a subscription to website B is £100 and it increases by £10 each year.

Which subscription is most expensive?

Challenge

In the first year, the value of an investment increases by 10%

In the next year, the value of the investment decreases by 10%

What percentage of the original amount remains?

Question 1

Josh invests £800 in a bank account for two years.

He receives compound interest of 3%

How much does he have in his account at the end of two years? **£848.72**

Question 2

The population of a country was 10 million at the start of 2010.

It has been increasing at a rate of 5.4% per year.

18 796 948.

What was the population at the start of 2022?

Question 3

The price of a subscription to website A is £100 and it increases by 8% each year.

The price of a subscription to website B is £100 and it increases by £10 each year.

Which subscription is most expensive?
Website A is cheaper for years 1 to 7.

Challenge

In the first year, the value of an investment increases by 10%

In the next year, the value of the investment decreases by 10%

What percentage of the original amount remains?   **99%**

Depreciation

A car was bought for £12000.

The price of the car depreciates by 20% per year.

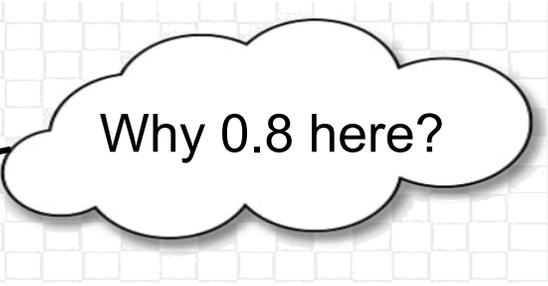What is its value after five years?

Depreciation

A car was bought for £12000.

The price of the car depreciates by 20% per year.

What is its value after five years?

Why 0.8 here?

$$= 12\ 000 \times 0.8^5$$

$$= £3932.16$$

1. A car is initially worth £8000.

   The value of the car decreases by 15% per year?

   What is the car worth after 4 years

2. The temperature in °C of a liquid decreases by 10°C every 10 minutes.

   What will the temperature be after $1\frac{1}{2}$ hours?

3. A phone battery loses 15% of its charge every hour.

   Assuming the phone is initially 100% charged, after how long will 50% charge remain?

   The phone powers down when 5% charge is left. How long will this take?

Challenge

The temperature of a cooling substance is modelled by the equation $\theta = 64 \times 0.5^t + 20$

θ = temperature in °C and t = time in minutes

What is the temperature when t = 0 ?

What will happen to temperature of the substance if it is left for a long time?

Why might this occur?

1. A car is initially worth £8000.

   The value of the car decreases by 15% per year?

   What is the car worth after 4 years **£4176.05**

2. A liquid is at a temperature of 42°C.
   The temperature of the liquid decreases by 10% every 10 minutes.

   What will the temperature be after $1\frac{1}{2}$ hours? **16.27°C**

3. A phone battery loses 15% of its charge every hour.

   Assuming the phone is initially 100% charged, after how long will 50% charge remain?   **5 hours**

   The phone powers down when 5% charge is left. How long will this take?
   19 hours

Challenge

The temperature of a cooling substance is modelled by the equation $\theta = 64 \times 0.5^t + 20$

   θ = temperature in °C and t = time in minutes

What is the temperature when t = 0 ? **84° C**
What will happen to temperature of the substance if it is left for a long time?   **Approach 20° C**
Why might this occur?   **Same as room temperature**

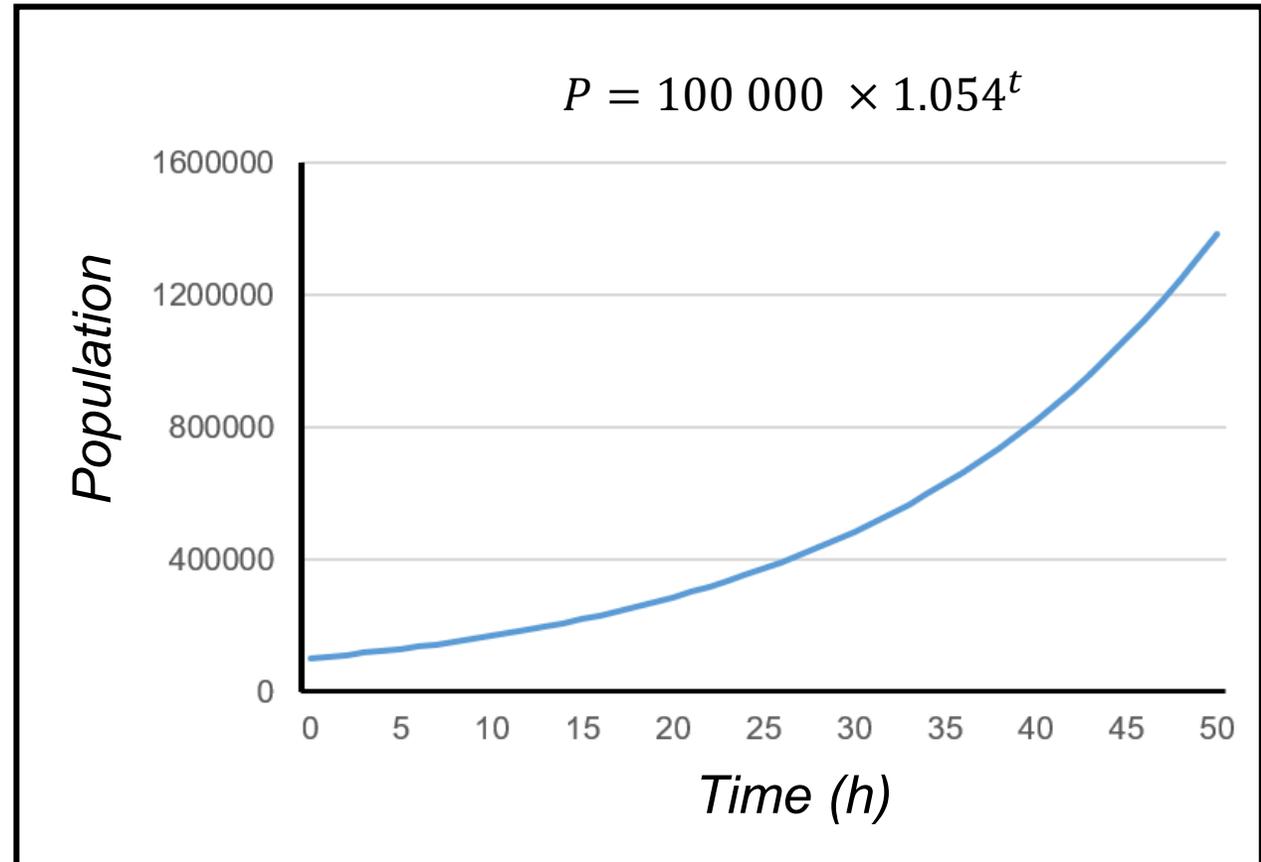# Review

Compound Growth

$$F = I \times r^n$$

F = final amount

I = Initial amount

r = increase factor

N = number of increases

Increases without limit.

$$P = 100\,000 \times 1.054^t$$

Population vs Time (h)

Review

Compound Reduction

$$F = I \times r^n$$

F = final amount

I = Initial amount

R = decrease factor

N = number of decreases

Decreases towards zero.

$$P = 100\,000 \times 0.946^t$$

Time (mins)
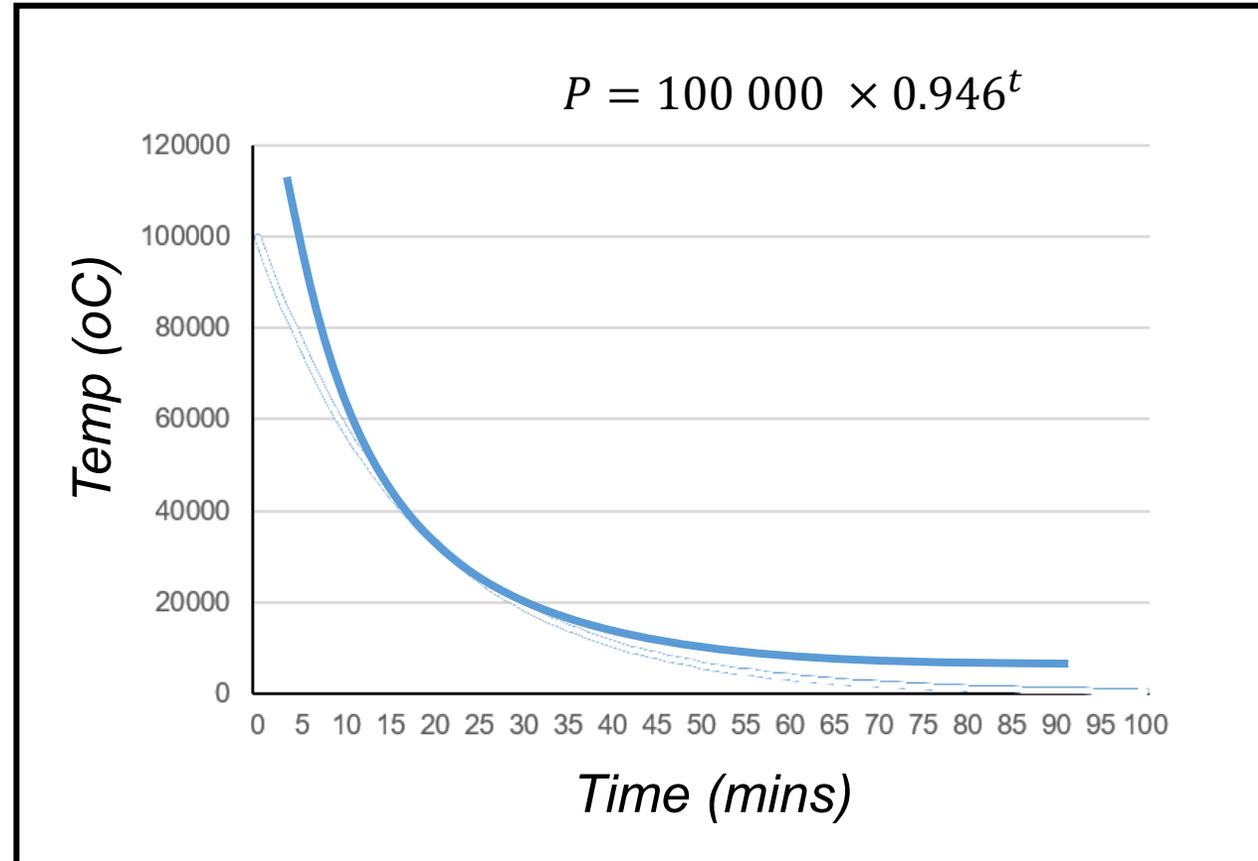
Extension

Investigate what happens when you evaluate this expression for n = 1 , 2 , 3 , 4 , …...

$$\left(1 + \frac{1}{n}\right)^n$$

Beth invests £100 000 in a savings account for 4 years.
The account pays compound interest at a rate of 2.5% per annum.

Calculate the total amount of money Beth will have in the account at the end of 4 years.

Solution

Beth invests 100 000 in a savings account for 4 years.
The account pays compound interest at a rate of 2.5% per annum.

Calculate the total amount of money Beth will have in the account at the end of 4 years.

$£100\ 000 \times (1.025)^4$

$= £110\ 381.29$

www.plexmaths.com