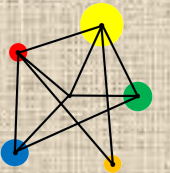


Translations



Recognise and perform translations of shapes.

Use a column vector to describe a translation.

Vocabulary

Translation

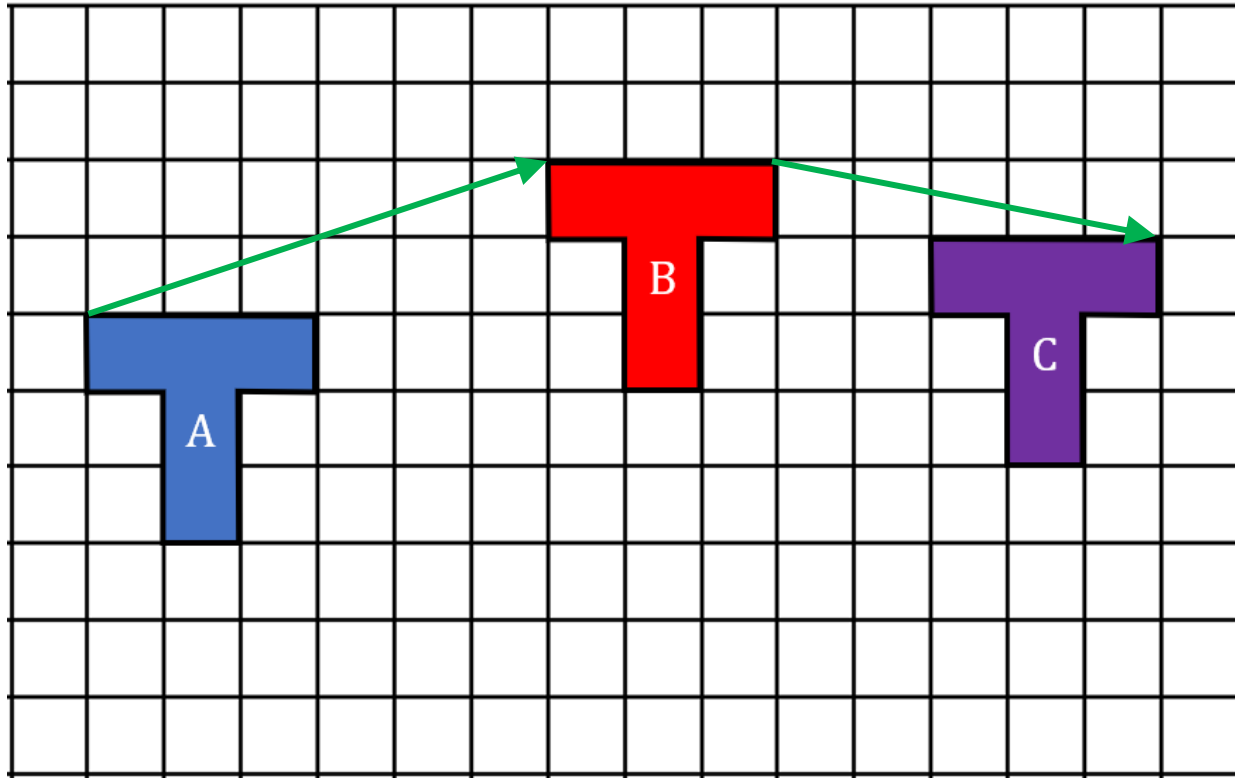
A movement of a shape to a different position.

Vector

A mathematical object that describes a translation.

Key Fact

A translation is a movement of a shape



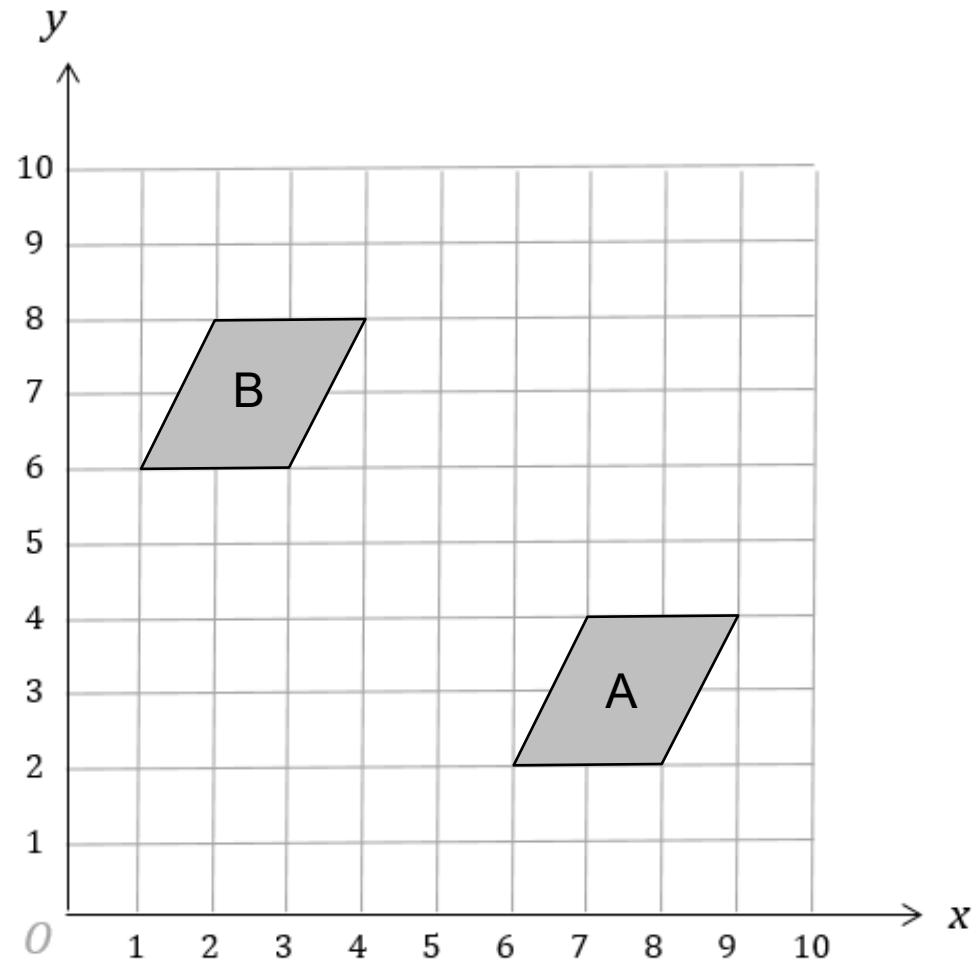
A translation is described by a **column vector**.

$$\overrightarrow{AB} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

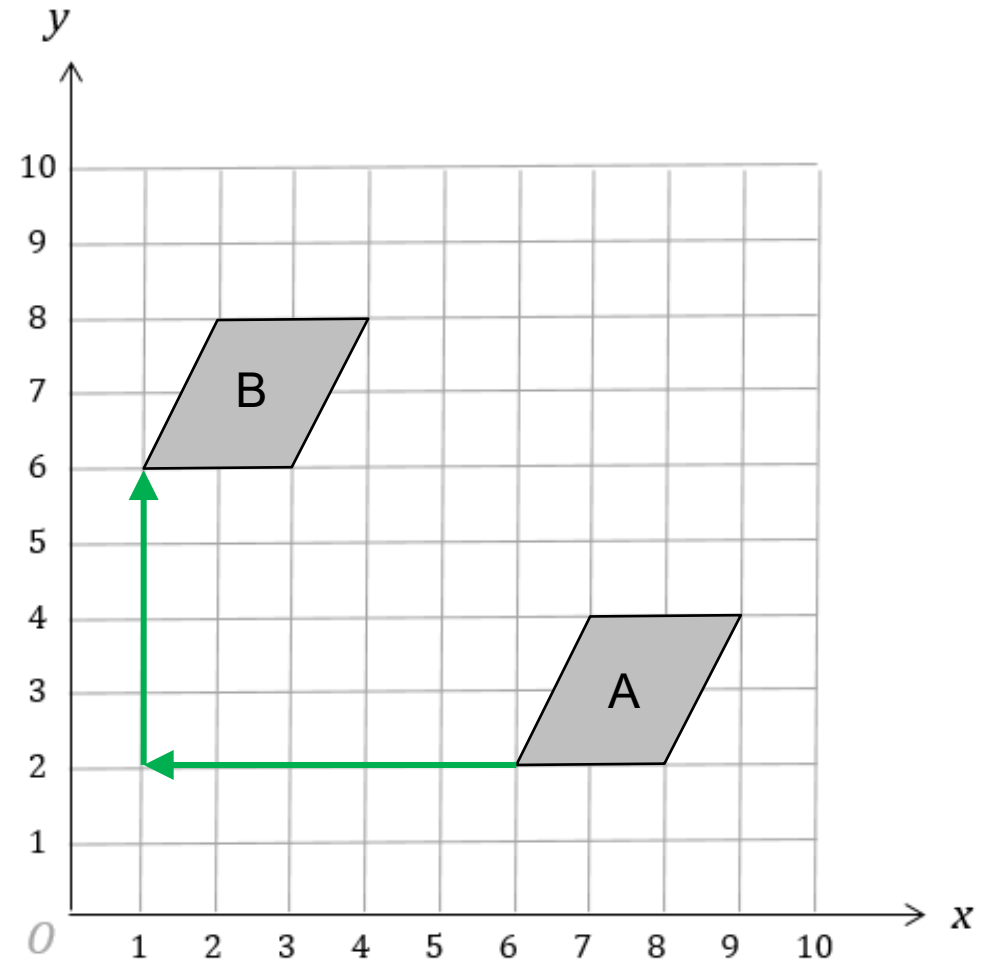
Example

Describe fully the transformation that maps shape A onto shape B



Solution

A translation by $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$



Example

Write down a column vector to describe the translation of shape A to the following shapes.

$$\overrightarrow{AB} =$$

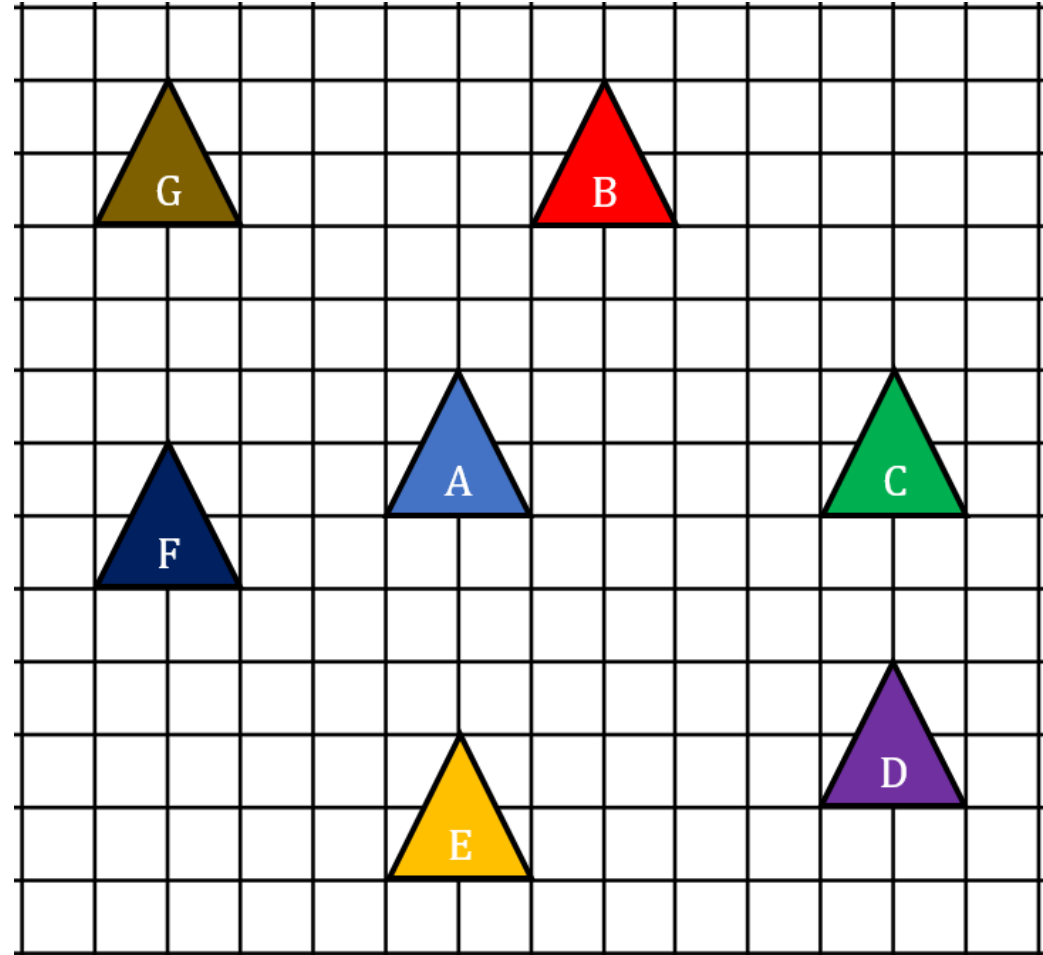
$$\overrightarrow{AC} =$$

$$\overrightarrow{AD} =$$

$$\overrightarrow{AE} =$$

$$\overrightarrow{AF} =$$

$$\overrightarrow{AG} =$$

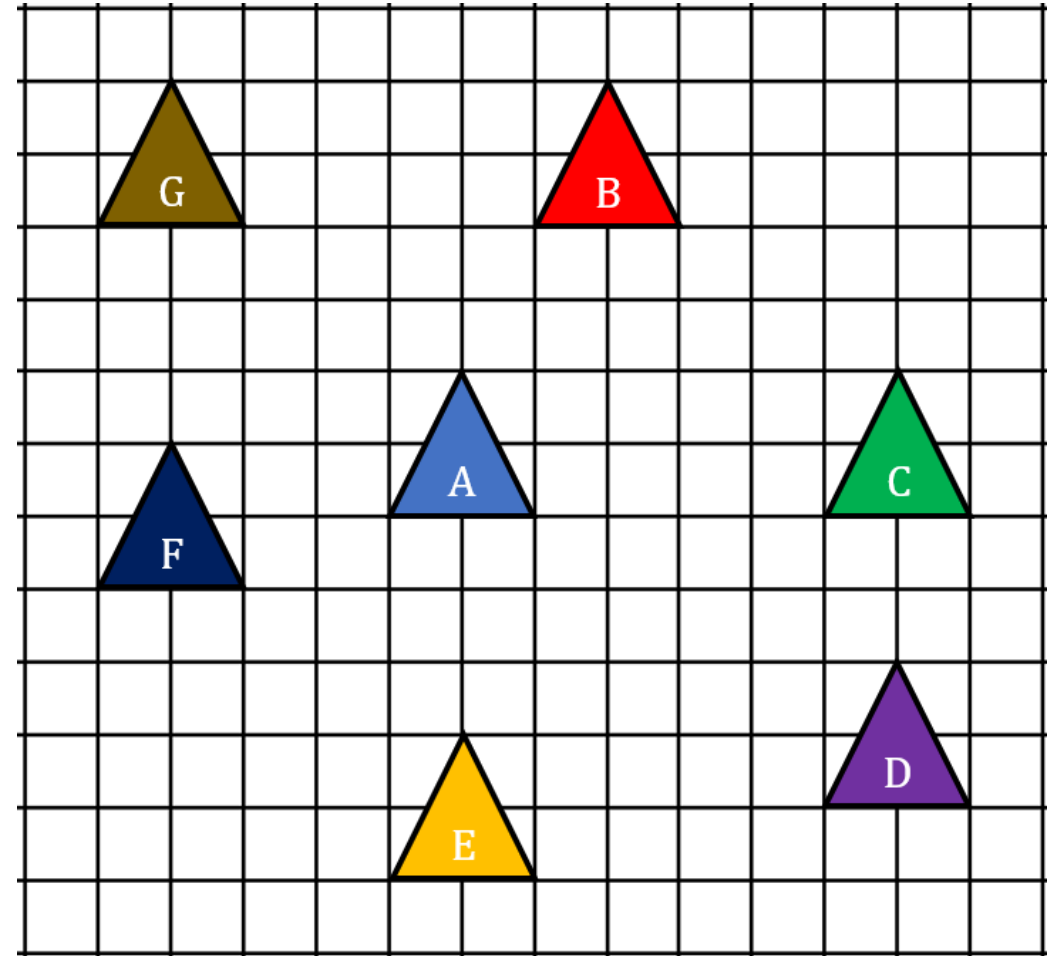


Solutions

$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad \overrightarrow{AC} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

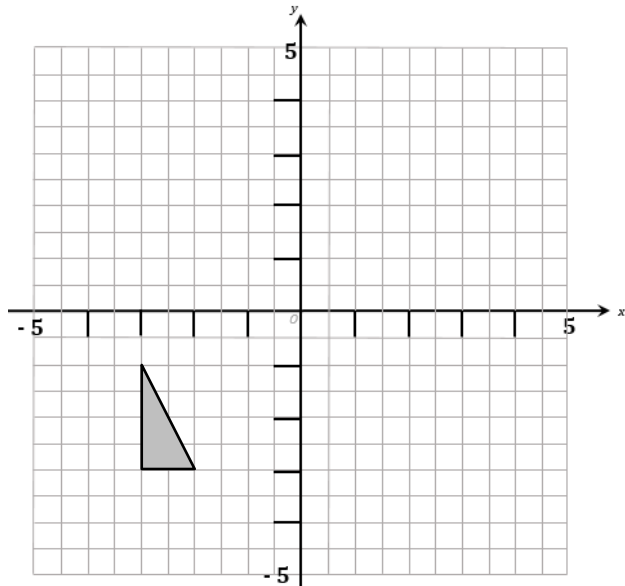
$$\overrightarrow{AD} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \quad \overrightarrow{AE} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$$

$$\overrightarrow{AF} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \quad \overrightarrow{AG} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$$

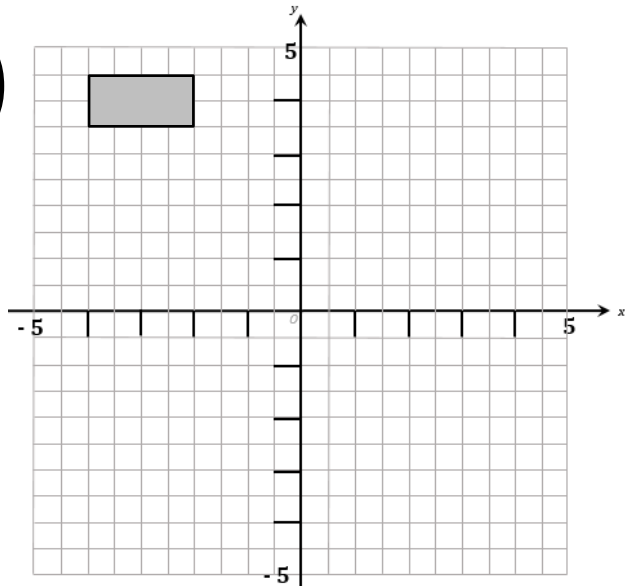


In each diagram, translate the shape using the given column vector.

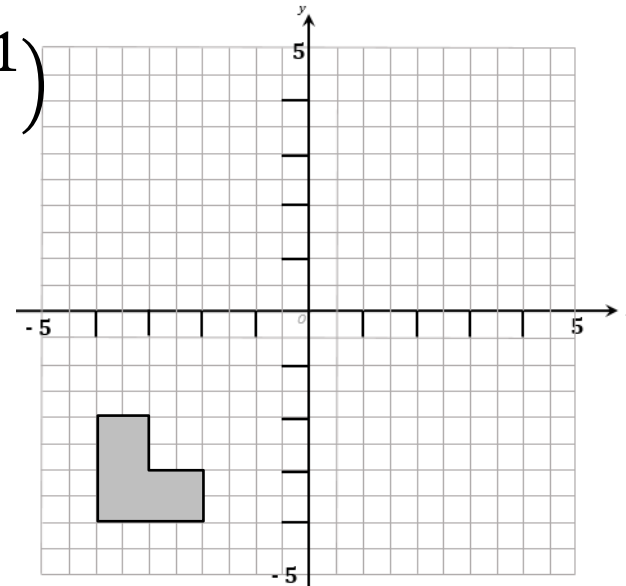
a) $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$



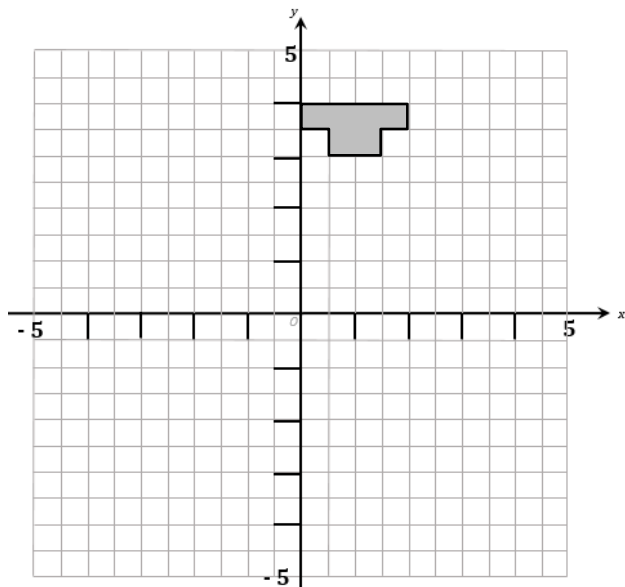
b) $\begin{pmatrix} 1 \\ -7 \end{pmatrix}$



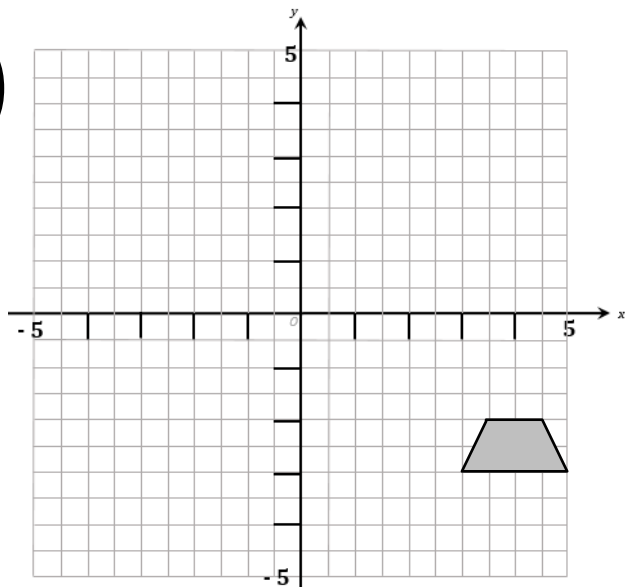
c) $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$



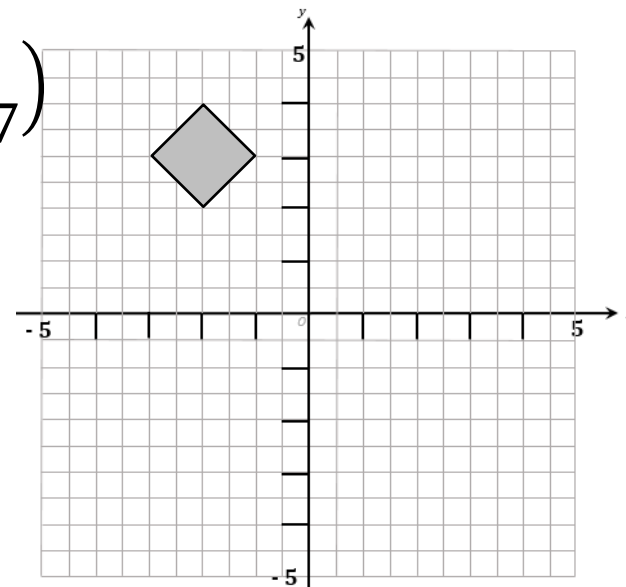
d) $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$



e) $\begin{pmatrix} -6 \\ 6 \end{pmatrix}$

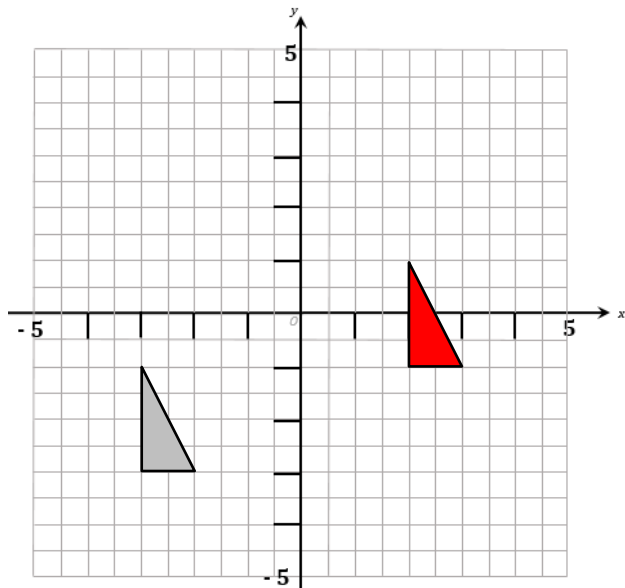


f) $\begin{pmatrix} 1 \\ -7 \end{pmatrix}$

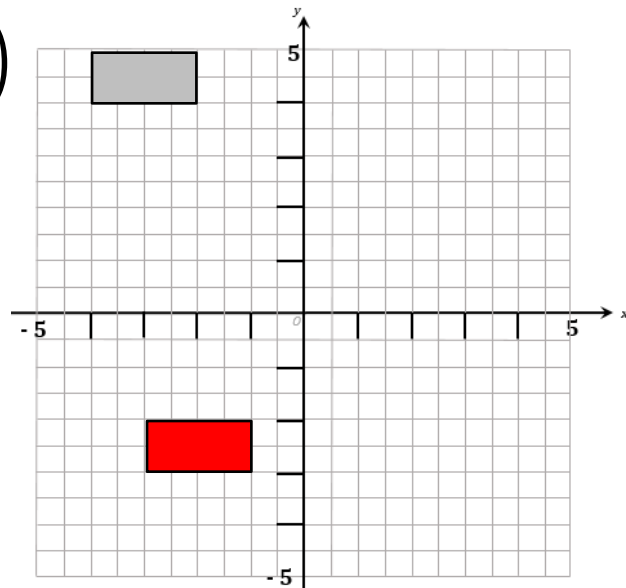


In each diagram, translate the shape using the given column vector.

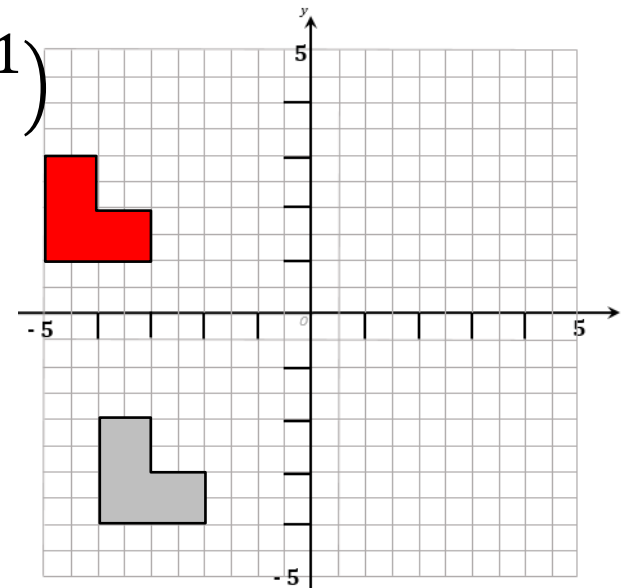
a) $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$



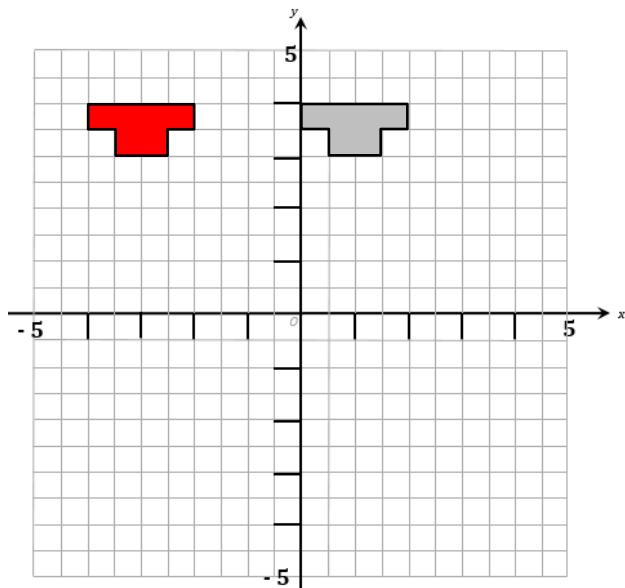
b) $\begin{pmatrix} 1 \\ -7 \end{pmatrix}$



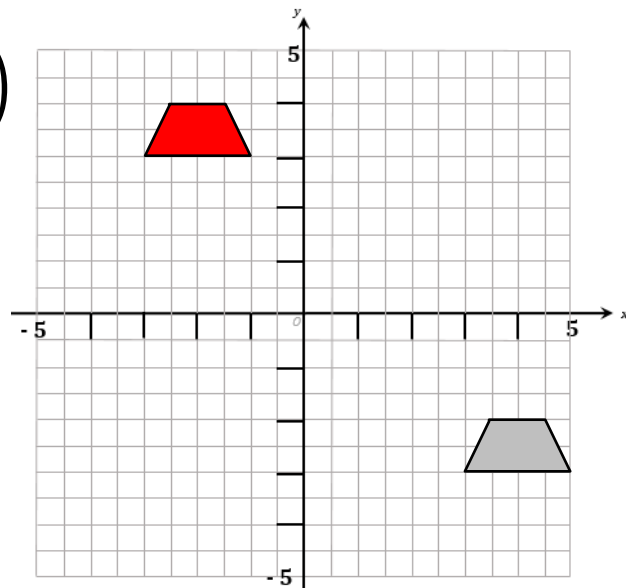
c) $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$



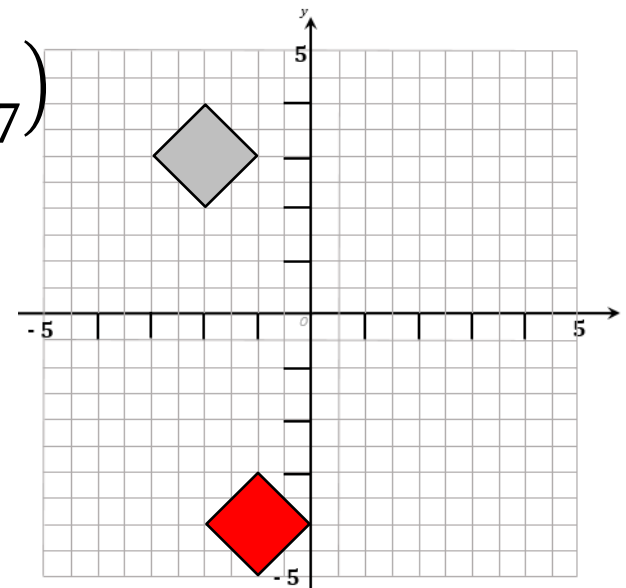
d) $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$



e) $\begin{pmatrix} -6 \\ 6 \end{pmatrix}$



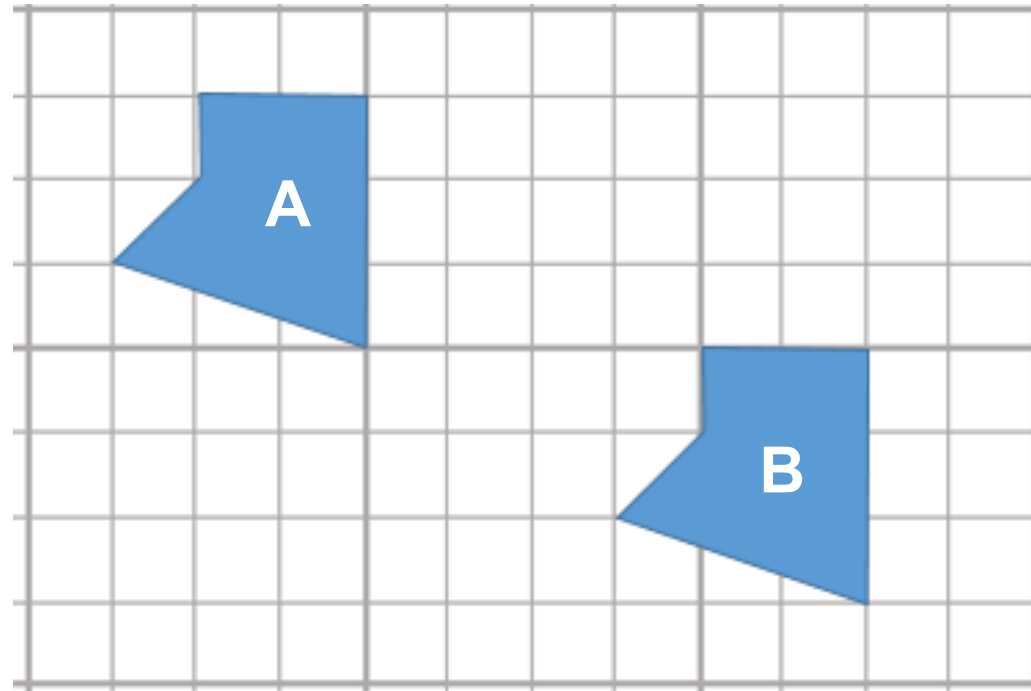
f) $\begin{pmatrix} 1 \\ -7 \end{pmatrix}$



Question

The translation \overrightarrow{AB} can be represented by the column vector $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$

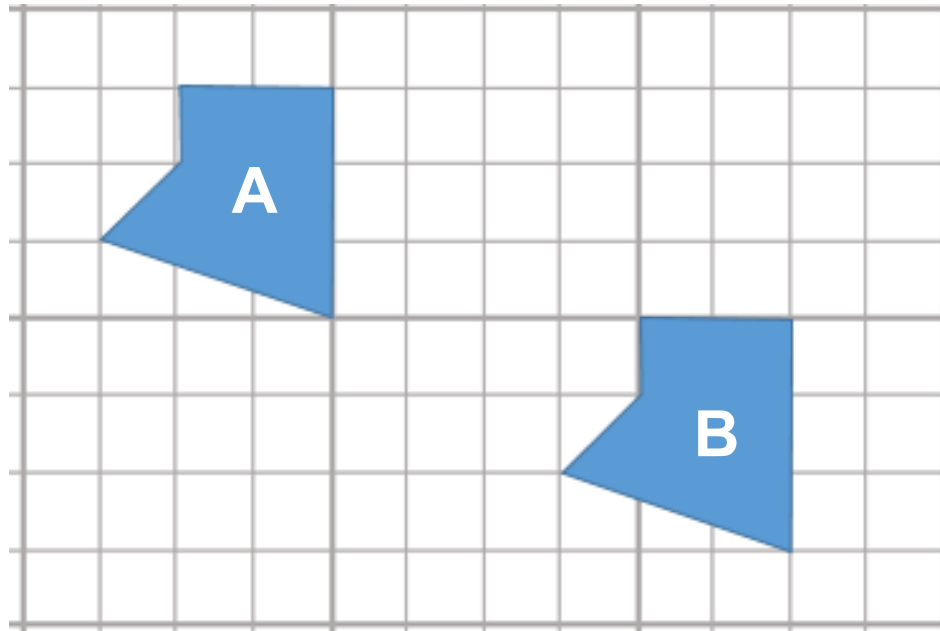
What is the **inverse** of the translation \overrightarrow{AB} ?



Solution

The translation \overrightarrow{AB} can be represented by the column vector $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$

What is the **inverse** of the translation \overrightarrow{AB} ?



The inverse translation of \overrightarrow{AB} is a translation described by the column vector $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$.

Key Fact

The inverse of translation $\begin{pmatrix} a \\ b \end{pmatrix}$ is the translation $\begin{pmatrix} -a \\ -b \end{pmatrix}$.

You try...

Can you find a column vector to describe the inverse translation to each of the translations below:.

$$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$\overrightarrow{EF} = \begin{pmatrix} -5 \\ -5 \end{pmatrix}$$

$$\overrightarrow{CD} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Solutions

Can you find a column vector to describe the inverse translation to each of the translations below:.

$$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

$$\overrightarrow{EF} = \begin{pmatrix} -5 \\ -5 \end{pmatrix} \quad \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

$$\overrightarrow{CD} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

Combined Translations

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

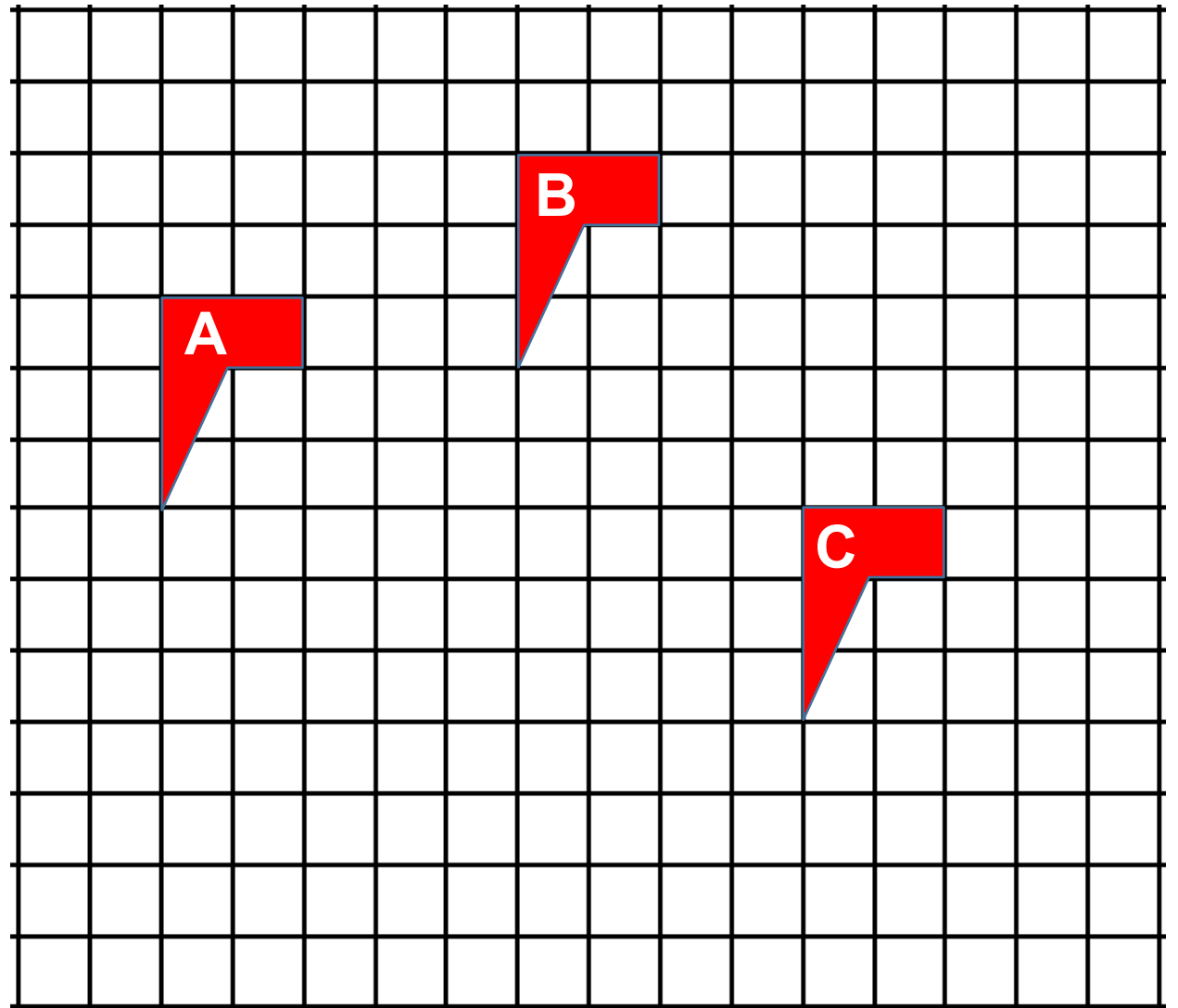
Write down the vectors:

i) \overrightarrow{AC}

ii) $2\overrightarrow{AB}$

iii) \overrightarrow{CB}

iv) \overrightarrow{CA}



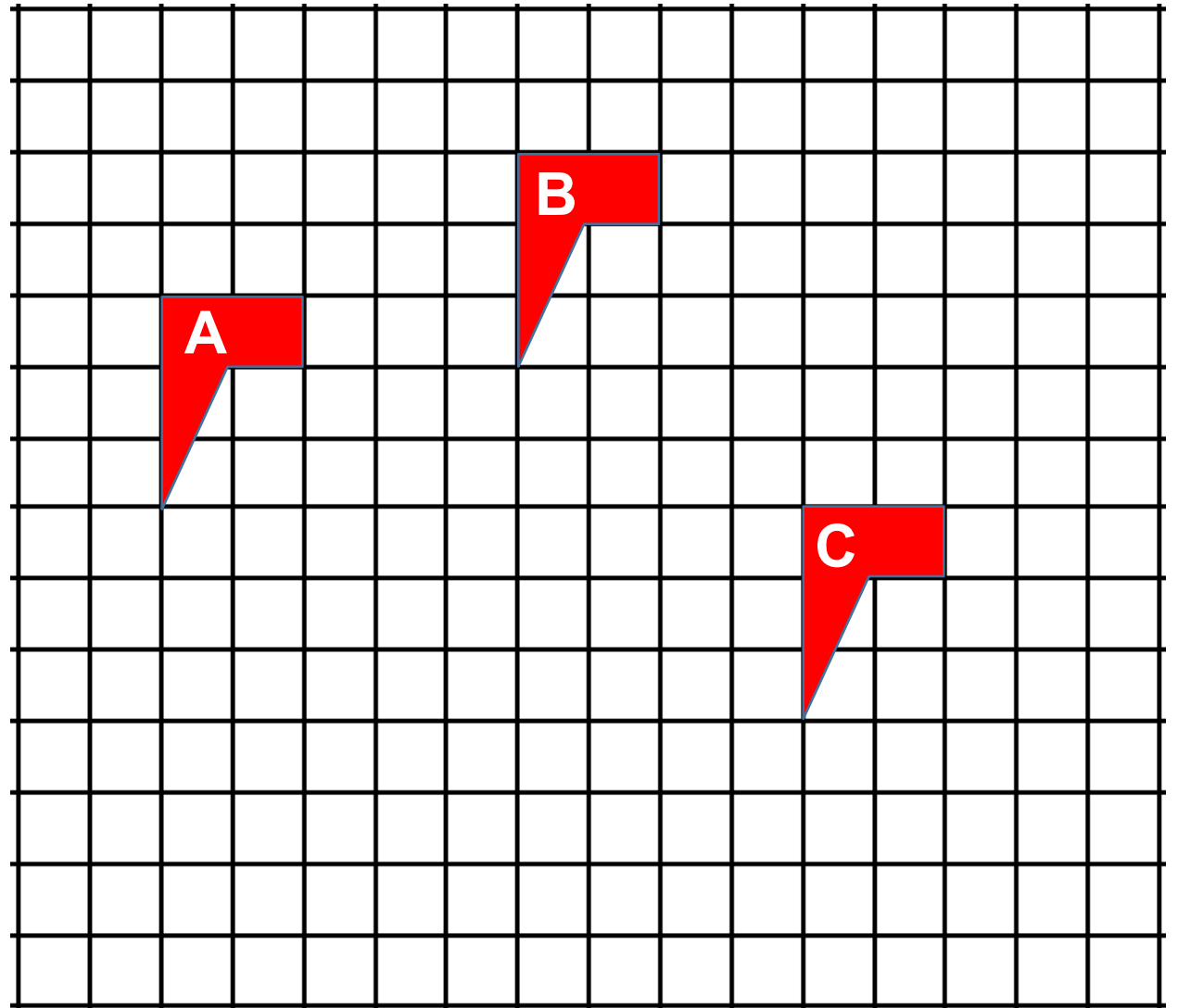
Solutions

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \quad \overrightarrow{BC} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

Write down the vectors:

i) $\overrightarrow{AC} = \begin{pmatrix} 9 \\ -3 \end{pmatrix}$ ii) $2\overrightarrow{AB} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$

iii) $\overrightarrow{CB} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$ iv) $\overrightarrow{CA} = \begin{pmatrix} -9 \\ 3 \end{pmatrix}$



Key Fact

- Vectors can be added. $\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a + c \\ b + d \end{pmatrix}$
- Vectors can be subtracted. $\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a - c \\ b - d \end{pmatrix}$
- Vectors multiplied by a scalar. $k \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} kc \\ kd \end{pmatrix}$

$$a = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \quad b = \begin{pmatrix} -2 \\ -4 \end{pmatrix} \quad c = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

Write down the column vectors representing:

1) $a + b$

2) $b + c$

3) $a - b$

4) $2a$

5) $2a + 3b$

6) $-c$

Challenge Write down 5 different vectors which have a sum of $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Solutions

$$a = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \quad b = \begin{pmatrix} -2 \\ -4 \end{pmatrix} \quad c = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

Write down the column vectors representing:

$$1) \quad a + b = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$2) \quad b + c = \begin{pmatrix} 2 \\ -9 \end{pmatrix}$$

$$3) \quad a - b = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

$$4) \quad 2a = \begin{pmatrix} 6 \\ 12 \end{pmatrix}$$

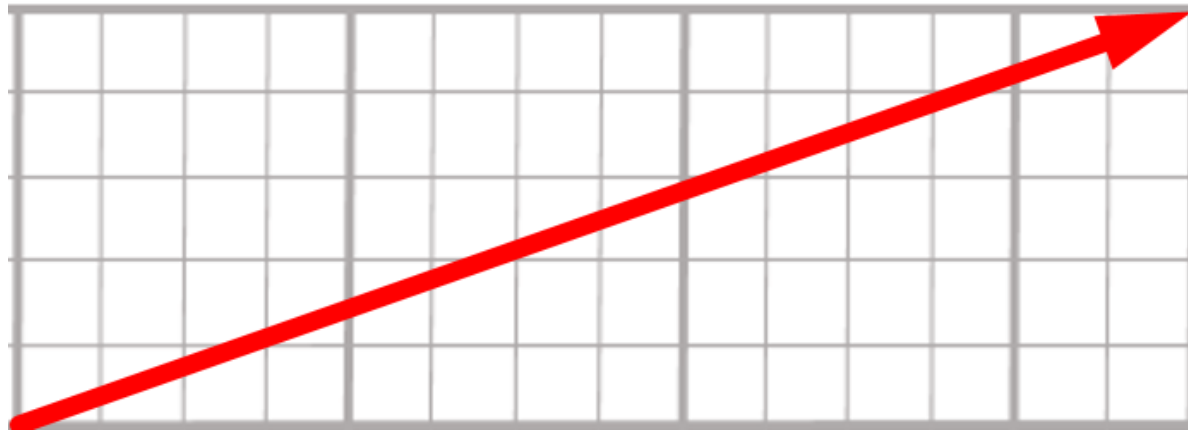
$$5) \quad 2a + 3b = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$6) \quad -c = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

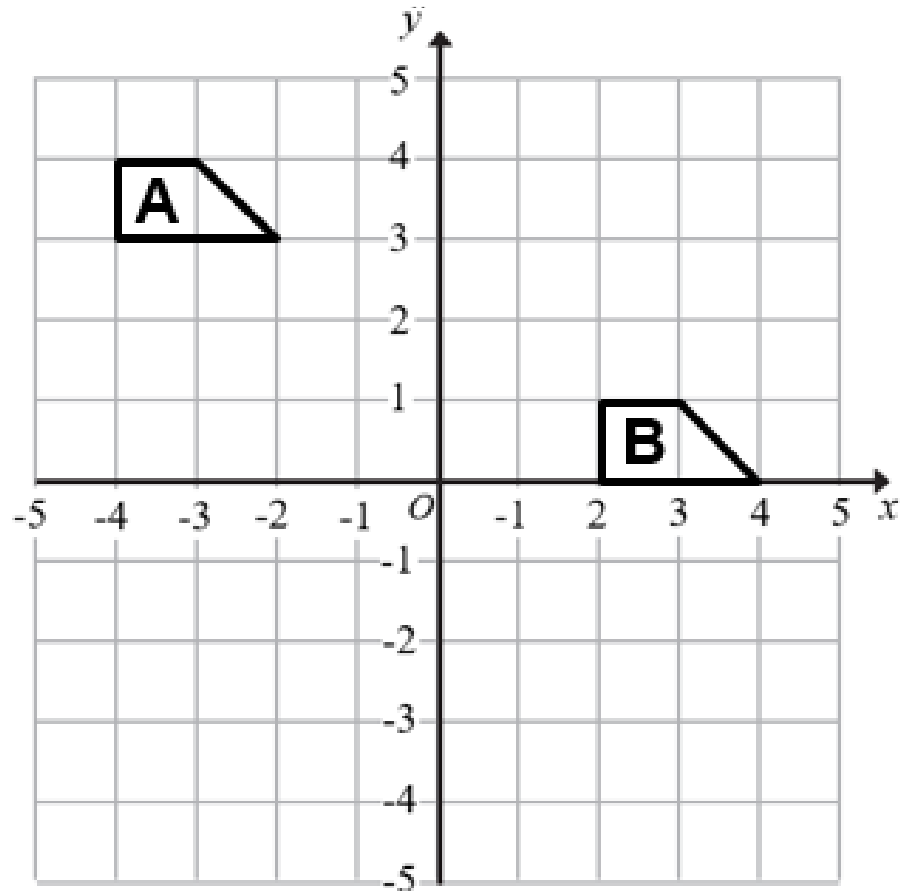
Challenge Write down 5 different vectors which have a sum of $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Extension

How could you find the length of the vector $\begin{pmatrix} 12 \\ 5 \end{pmatrix}$?

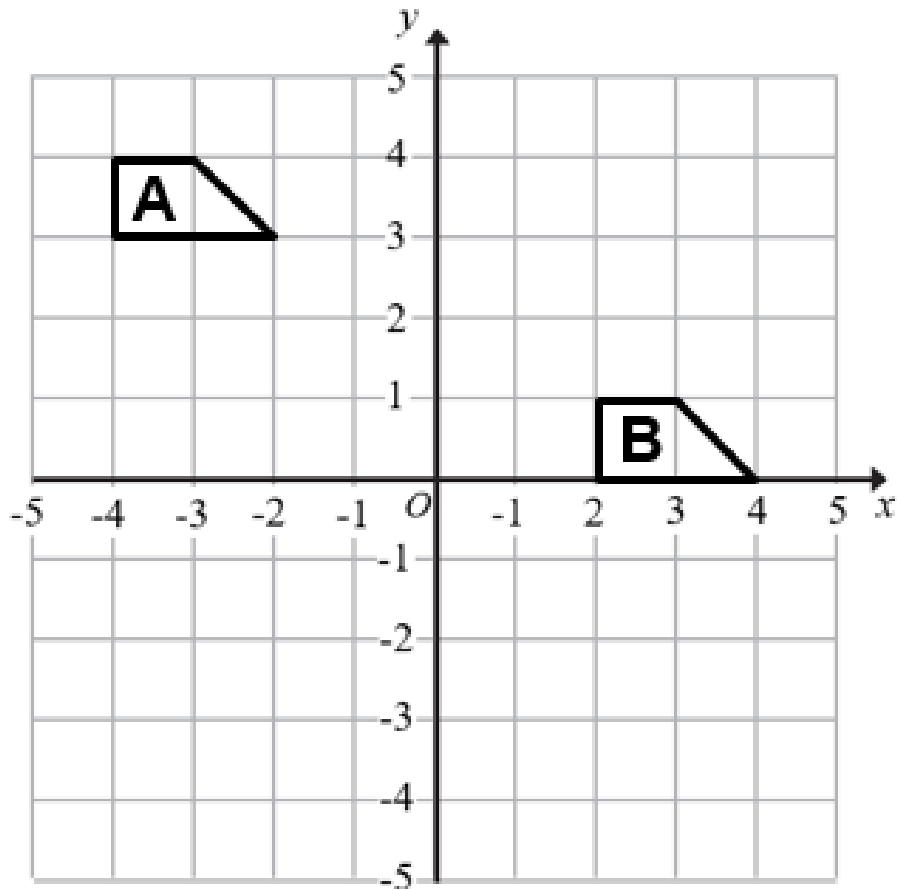


Exam Style Question



Describe the single transformation that maps shape A onto shape B.

Solution



Describe the single transformation that maps shape A onto shape B.

Translation with vector $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$

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