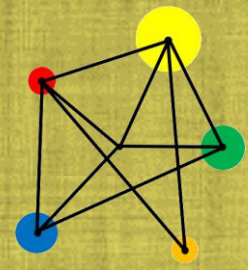
Algebra: Substitution



Substitute positive numbers, negative numbers, fractions and decimals into expressions and formulae.

Vocabulary

Expression
A collection of letters and numbers.

Formula
Similar to an expression, a formula must have a subject.

Substitute
Replace letters in an expression or formulae with numerical values.

Review : Order of Operation

Evaluate:

a) 5×6

b) $5 \times 6 + 2$

c) $5 + 6 \times 2$

d) $(5 + 6) \times 2$

e) $10 + 4 \div 2$

f) 5^2

g) 2^5

h) $\frac{6 + 2}{4}$

i) $\sqrt{\frac{5^2 + 3^2 + 4^3}{2}}$

j) $-5 + 7$

k) $-4 - 6$

l) -5×-6

Solutions

Evaluate:

a) 5×6

= 30

b) $5 \times 6 + 2$

= 32

c) $5 + 6 \times 2$

= 17

d) $(5 + 6) \times 2$

= 22

e) $10 + 4 \div 2$

= 12

f) 5^2

= 25

g) 2^5

= 32

h) $\frac{6 + 2}{4}$

= 2

i) $\sqrt{\frac{5^2 + 3^2 + 4^3}{2}}$

= 7

j) $-5 + 7$

= 2

k) $-4 - 6$

= - 10

l) -5×-6

= 30

Key Facts

An **expression** is a collection of letters and numbers.

Expressions

$3x - y^2 + 5$

$a + b + c$

$\left(\frac{c}{\sqrt{a^2 + b^2}}\right)^{\frac{3}{2}}$

A **formula** is like an expression but it must have a **subject**.

Formulae

$A = \pi r^2$

$F = \frac{9C}{5} + 32$

$s = ut + \frac{1}{2}at^2$

Quick Check

Which of these are expressions and which are formulae?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{1}{2}(A + B)$$

$$\%\, profit = \frac{profit}{original\ amount} \times 100$$

$$\frac{4}{3}\pi r^3$$

Solutions

Which of these are expressions and which are formulae?

Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Expression

$$\frac{1}{2}(A + B)$$

Formula

$$\%\, profit = \frac{profit}{original\ amount} \times 100$$

Expression

$$\frac{4}{3}\pi r^3$$

Examples

a = 2 , b = 3 , c = 5

Work out the value of each expression:

1. a + b + c

2. ab

3. $\frac{2b + 3a}{2c}$

4. $\sqrt{c^2 - b^2}$

Challenge

Evaluate

$$\frac{abc}{cba}$$

Solutions

a = 2 , b = 3 , c = 5

Work out the value of each expression:

1. a + b + c = 10

2. ab = 6

3. $\frac{2b + 3a}{2c}$ = 1.2

4. $\sqrt{c^2 - b^2}$ = 4

Challenge

Evaluate

$\frac{abc}{cba}$ = 1

Exercise

a = 3 , b = 2 , c = 4

Evaluate:

1. a + b + c
2. 2b - c
3. ab - c
4. b – 2a
5. ba
6. abc

a = 40 , b = 5 , c = 8

Evaluate:

1. a - bc
2. $a^2 + b^2$
3. $\frac{a}{b}$
4. $\frac{a}{bc}$
5. 2(b + c)
6. $\frac{a}{10}$ (b - c)

a = 3 , b = 4, c = $\frac{1}{2}$

Evaluate:

1. $\sqrt{a^2 + b^2}$
2. $\frac{b}{c}$
3. $\frac{ab}{c}$
4. c^3
5. $\frac{c^2}{b}$

Solutions

a = 3 , b = 2 , c = 4

Evaluate:

1. a + b + c = **9**
2. 2b - c = **0**
3. ab - c = **2**
4. b – 2a = **- 4**
5. ba = **6**
6. abc = **24**

a = 40 , b = 5 , c = 8

Evaluate:

1. a - bc = **0**
2. $a^2 + b^2$ = **1625**
3. $\frac{a}{b}$ = **8**
4. $\frac{a}{bc}$ = **1**
5. 2(b + c) = **26**
6. $\frac{a}{10}$ (b - c) = **- 12**

a = 3 , b = 4, c = $\frac{1}{2}$

Evaluate:

1. $\sqrt{a^2 + b^2}$ = **5**
2. $\frac{b}{c}$ = **8**
3. $\frac{ab}{c}$ = **24**
4. c^3 = $\mathbf{\frac{1}{8}}$
5. $\frac{c^2}{b}$ = $\mathbf{\frac{1}{16}}$

x = 2 , y = 4 , z = 10

Extra Exercise

Work out:

$x + y$	$2z$	$x - y$	xy	yzx
$\frac{z}{5}$	$\frac{8}{2x}$	y^2	x^5	$z \div x$
$xy + yz$	$x(y + z)$	$z(z + 1)$	$\frac{x + z}{y}$	$\sqrt{x + y + z}$
$\frac{z}{x}$	$\frac{x}{z}$	$\sqrt[3]{yx}$	xz^2	$(xz)^2$

x = 2 , y = 4 , z = 10

Solutions

Work out:

$x + y = 6$ $2z = 20$ $x - y = -2$ $xy = 8$ $yzx = 80$

$\frac{z}{5} = 2$ $\frac{8}{2x} = 2$ $y^2 = 16$ $x^5 = 32$ $z \div x = 5$

$xy + yz = 48$ $x(y + z) = 28$ $z(z + 1) = 110$ $\frac{x + z}{y} = 3$ $\sqrt{x + y + z} = 4$

$\frac{z}{x} = 5$ $\frac{x}{z} = \frac{1}{5}$ $\sqrt[3]{yx} = 2$ $xz^2 = 200$ $(xz)^2 = 400$

Substituting Negative Numbers

$a = 5$, $b = -2$, $c = -8$

Evaluate:

1. $a + b$
2. $b - a$
3. $a - b$
4. ab
5. bc
6. a^3

$a = 5$, $b = -2$, $c = -8$

Evaluate:

1. abc
2. c^2
3. $12 - ab$
4. $a^3 - b^3$
5. $\sqrt{bc}$

$d = 8$, $e = \frac{3}{4}$, $f = -\frac{1}{2}$

Evaluate:

1. ed
2. $e + f$
3. ef
4. $\frac{f}{e}$
5. f^{10}

Solutions

$a = 5$, $b = -2$, $c = -8$

Evaluate:

1. $a + b = 3$
2. $b - a = -7$
3. $a - b = 7$
4. $ab = -10$
5. $bc = 16$
6. $a^3 = 125$

$a = 5$, $b = -2$, $c = -8$

Evaluate:

1. $abc = 80$
2. $c^2 = 64$
3. $12 - ab = 22$
4. $a^3 - b^3 = 133$
5. $\sqrt{bc} = 4$

$d = 8$, $e = \frac{3}{4}$, $f = -\frac{1}{2}$

Evaluate:

1. $ed = 6$
2. $e + f = \frac{1}{4}$
3. $ef = -\frac{3}{8}$
4. $\frac{f}{e} = -\frac{2}{3}$
5. $f^{10} = \frac{1}{1024}$

Formulae : Examples

A is the **subject** of the **formula** below.

$$A = \frac{(a+b)h}{2}$$

Evaluate A when a = 8 , b = 12 and h = 5

Solution

A is the **subject** of the **formula** below.

$$A = \frac{(a+b)h}{2}$$

Evaluate A when a = 8 , b = 12 and h = 5

$$A = \frac{(8 + 12) \times 5}{2}$$

$$= \frac{20 \times 5}{2}$$

$$= \frac{100}{2} \qquad = \mathbf{50}$$

Substitute the given values into each formula.

$A = \frac{(a+b)h}{2}$	$t = r^4 - 4r$	$m = d(e-f)$
$a = 12 \ , \ b = 1 \ , \ h = 4$	$r = 2$	$d = 4 \ , \ e = 4 \ , \ f = -1$
$P = \frac{Q}{R+S}$	$s = \frac{1}{2}(u+v)t$	$T = 6.28\sqrt{\frac{L}{g}}$
$Q = 8 \ , \ R = 0.1 \ , \ S = 0.3$	$u = 5 \ , \ v = 11 \ , \ t = 8$	$L = 1 \ , \ g = 9.81$
$R = \frac{1}{R_1} + \frac{1}{R_2}$	$z = \frac{x^2 - y^2}{x - y}$	
$R_1 = 1 \ , \ R_2 = 0.1$	$x = 5 \ , \ y = 3$	

Substitute the given values into each formula.

Solutions

$A = \frac{(a+b)h}{2}$ **$A = 26$** $a = 12$, $b = 1$, $h = 4$	$t = r^4 - 4r$ **$t = 8$** $r = 2$	$m = d(e - f)$ **$m = 20$** $d = 4$, $e = 4$, $f = -1$
$P = \frac{Q}{R+S}$ **$P = 20$** $Q = 8$, $R = 0.1$, $S = 0.3$	$s = \frac{1}{2}(u+v)t$ **$s = 64$** $u = 5$, $v = 11$, $t = 8$	$T = 6.28\sqrt{\frac{L}{g}}$ **$T = 2.005$** $L = 1$, $g = 9.81$
$R = \frac{1}{R_1} + \frac{1}{R_2}$ **$R = 11$** $R_1 = 1$, $R_2 = 0.1$	$z = \frac{x^2 - y^2}{x - y}$ **$z = 8$** $x = 5$, $y = 3$	

Extension

$y = 3x + 2$

Work out the value of x when y = 29

Exam Style Question

Here are four expressions.

$$\frac{x}{y} \qquad \frac{y}{x} \qquad xy \qquad \frac{x^2}{y^2}$$

When x = 10 and y = 0.1 which expression has the largest value?

You **must** show your working.

Solution

Here are four expressions.

$$\frac{x}{y} \qquad \frac{y}{x} \qquad xy \qquad \frac{x^2}{y^2}$$

When x = 10 and y = 0.1 which expression has the largest value?

You **must** show your working.

$$\frac{x}{y} = 100 \qquad \frac{y}{x} = 0.01 \qquad xy = 1 \qquad \frac{x^2}{y^2} = 10\,000$$

$\frac{x^2}{y^2}$ ***has the largest value.***

Exam Style Question

Michelle is trying to work out the two values of t for which $24 - t^3 = 16$

Her values are 2 and - 2

Are her values correct?

You **must** show your working.

Solution

Michelle is trying to work out the two values of t for which $24 - t^3 = 16$

Her values are 2 and - 2

Are her values correct?

You **must** show your working.

When t = 2:

$$24 - 2^3$$
$$= 24 - 8$$
$$= \underline{16}$$

When t = – 2:

$$24 - (-2)^3$$
$$= 24 - -8$$
$$= \underline{32}$$

Michelle is incorrect.