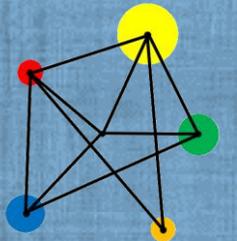# Product of Prime Factors

Write a number as a product of prime factors

Use product of prime factors to find HCF & LCM

Vocabulary

Product
A multiplication.
The product of 5 and 10 is 50

Factor
A number that divides exactly into
another number.
The factors of 21 are 1 , 3 , 7 and 21

Prime Number
A number that has exactly 2 factors

Integer
A whole number.

Warm up exercise

1. Write down the factors of 18.

2. Find the product of 6 and 4.

3. Is 1 a prime number?

4. Evaluate 2^3 x 3^2

Solutions

1. Write down the factors of 18.   **1 , 2 , 3 , 6 , 9 , 18**

2. Find the product of 6 and 4.   **6 x 4 = 24**

3. Is 1 a prime number?   **No. Prime numbers have exactly two factors and 1 has only one factor.**

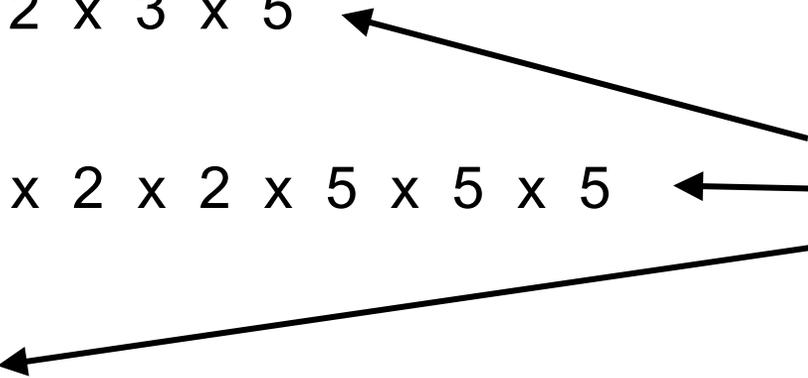4. Evaluate 2^3 x 3^2 **8 x 9 = 72**

Key Facts

Any integer can be written as a product of prime numbers.

60 = 2 x 2 x 3 x 5

1000 = 2 x 2 x 2 x 5 x 5 x 5   *All prime numbers*
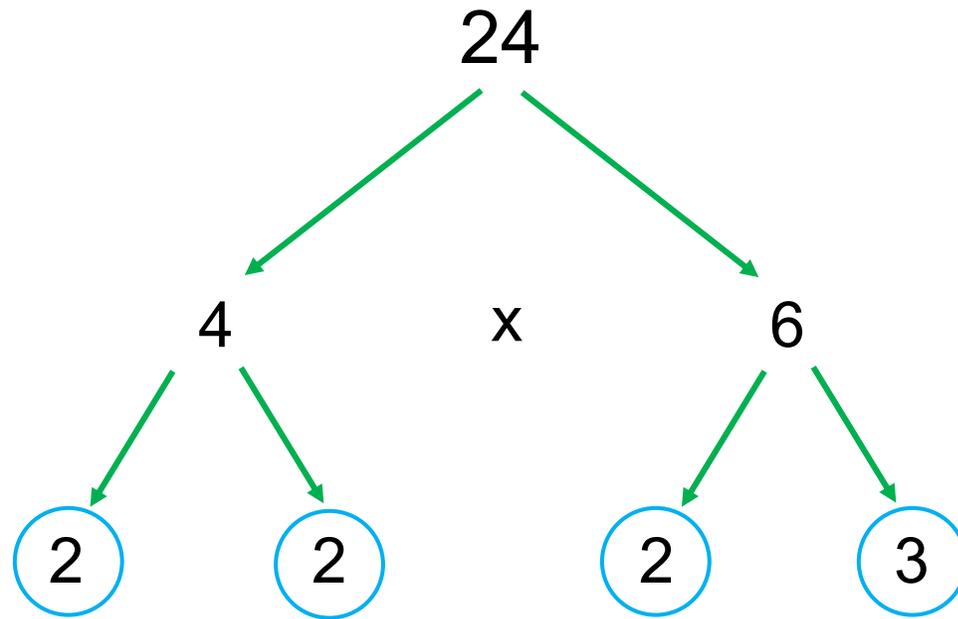
83 = 83

For any given integer, its product of prime factors will be unique.

Example

Write 24 as a product of prime factors.

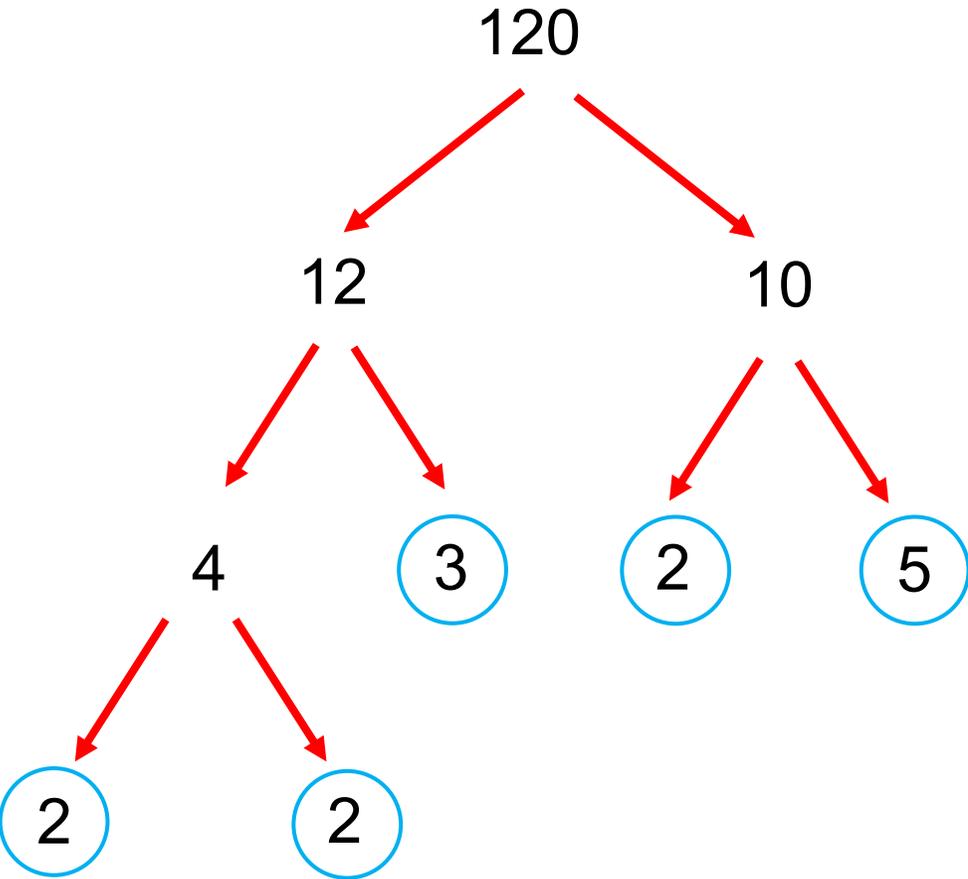# Solution

Write 24 as a product of prime factors.
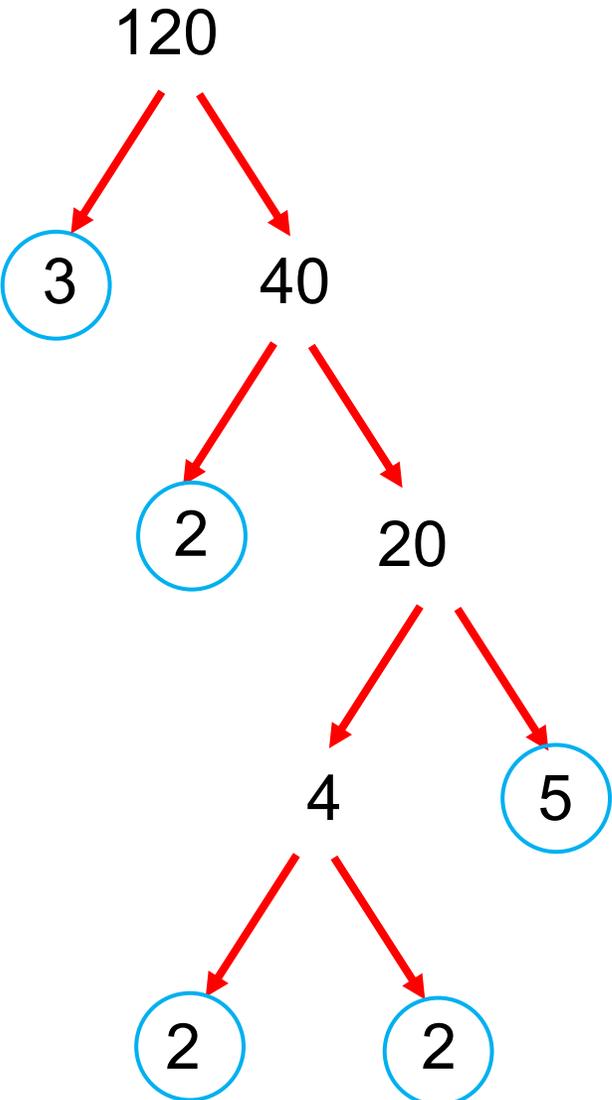
24

4   x   6

2   2   2   3

$24 = 2^3 \times 3$

You try ..

Write 120 as a product of prime factors.

Solution

120

3   40

2   20

4   5

2   2

120

12   10

4   3   2   5

2   2

$120 = 2^3 \times 3 \times 5$

Exercise

Write as a product of prime factors:

a) 10

b) 6
.

c) 9

d) 8

e) 12

f) 11.

Write as a product of prime factors:

a) 20

b) 40

c) 28

d) 100

e) 180

Write as a product of prime factors:

a) 96

b) 98

c) 221

d) 298

Challenge
Find two numbers which have a product of 10 000. Neither number contains any zeroes

Solutions

Write as a product of prime factors:

a) 10 $= 2 \times 5$

b) 6 $= 2 \times 3$

c) 9 $= 3 \times 3$

d) 8 $= 2^3$

e) 12 $= 2^2 \times 3$

f) 11 $= 11$

Write as a product of prime factors:

a) 20 $= 2^2 \times 5$

b) 40 $= 2^3 \times 5$

c) 28 $= 2^2 \times 7$

d) 100 $= 2^2 \times 5^2$

e) 180 $= 2^2 \times 3^2 \times 5$

Write as a product of prime factors:

a) 96 $= 2^5 \times 3$

b) 98 $= 2 \times 7^2$

c) 221 $= 13 \times 17$

d) 298 $= 2 \times 149$

Challenge
Find two numbers which have a product of 10 000. Neither number contains any zeroes $16 \times 625 = 10\ 000$

Extra Exercise

Write each number as a product of prime factors.

a) 54

b) 225

c) 216

d) 18

e) 100

f) 1 024

g) 111

h) 101

i) 1 000 000

j) 2310

k) 3375

l) 1331

Challenge

Write 1739 as a product of prime factors

Solutions

Write each number as a product of prime factors.

a) 54 **= 2 x 3³**

b) 225 **= 3² x 5²**

c) 216 **= 2³ x 3³**

d) 18 **= 2 x 3²**

e) 100 **= 2² x 5²**

f) 1 024 **= 2¹⁰**

g) 111 **= 3 x 37**

h) 101 **= 101**

i) 1 000 000 **= 2⁶ x 5⁶**

j) 2310 **= 2 x 3 x 5 x 7 x 11**

k) 3375 **= 3³ x 5³**

l) 1331 **= 11³**

Challenge **37 x 47**

Write 1739 as a product of prime factors

Key Fact

We can use products of prime factors to help us find:

     i) Highest common factor.

    ii) Lowest common multiple.

Example

a) Find the highest common factor (HCF) of 8 and 12

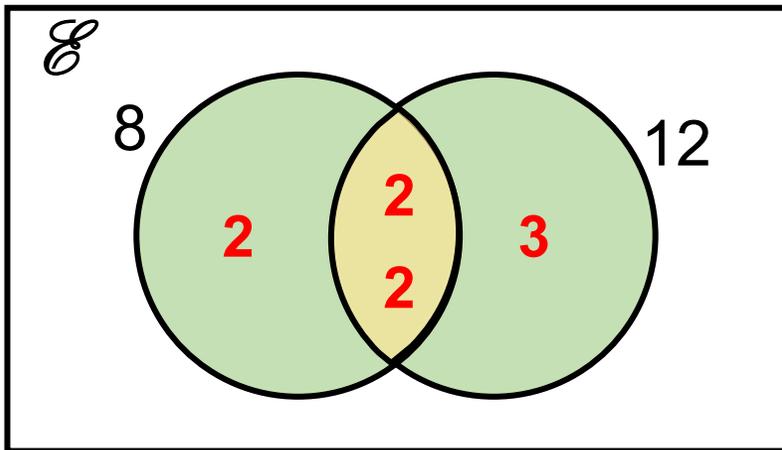b) Find the lowest common multiple (LCM) of 8 and 12

Solution

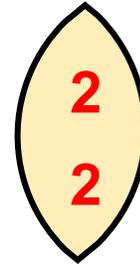1. Write each number as a product of prime factors.

   $$8 = 2 \times 2 \times 2$$
   $$12 = 2 \times 2 \times 3$$

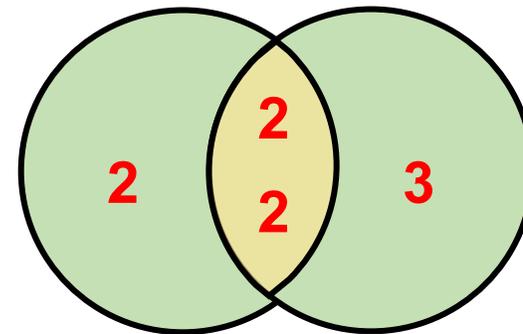2. Construct a Venn diagram.

3. HCF is the product of numbers in the intersection.

   $$HCF = 2 \times 2 = \underline{4}$$

4. LCM is the product of all the numbers in the union.

   $$LCM = 2 \times 2 \times 2 \times 3 = \underline{24}$$

Exercise

Find the HCF and LCM of each pair of numbers:

a) 6 , 8

b) 2 , 4

c) 6 , 9

d) 8 , 12

e) 10 , 15

f) 4 , 18.

Find the HCF and LCM of each pair of numbers:

a) 30 , 42

b) 30 , 80

c) 75 , 105

d) 132 , 187

e) 36 , 45 , 72

Find the HCF and LCM of each set of numbers:

a) 104 , 286

b) 198 , 207

c) 48 , 128 , 240

d) 735 , 1617

Solutions

Find the HCF and LCM of each pair of numbers:

a) 6 , 8
   HCF = 2, LCM = 24

b) 2 , 4
   HCF = 2, LCM = 4

c) 6 , 9
   HCF = 3, LCM = 18

d) 8 , 12
   HCF = 4, LCM = 24

e) 10 , 15
   HCF = 5, LCM = 30

f) 4 , 18.
   HCF = 2, LCM = 36

Find the HCF and LCM of each pair of numbers:

a) 30 , 42
   HCF = 6, LCM = 210

b) 30 , 80
   HCF = 10, LCM = 240

c) 75 , 105
   HCF = 15, LCM = 525

d) 132 , 187
   HCF = 11, LCM = 2 244

e) 36 , 45 , 72
   HCF = 9, LCM = 360

Find the HCF and LCM of each set of numbers:

a) 104 , 286
   HCF = 26
   LCM = 1 144

b) 198 , 207
   HCF = 9
   LCM = 4 554

c) 48 , 128 , 240
   HCF = 16
   LCM = 1920

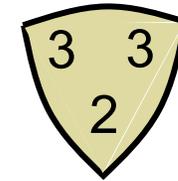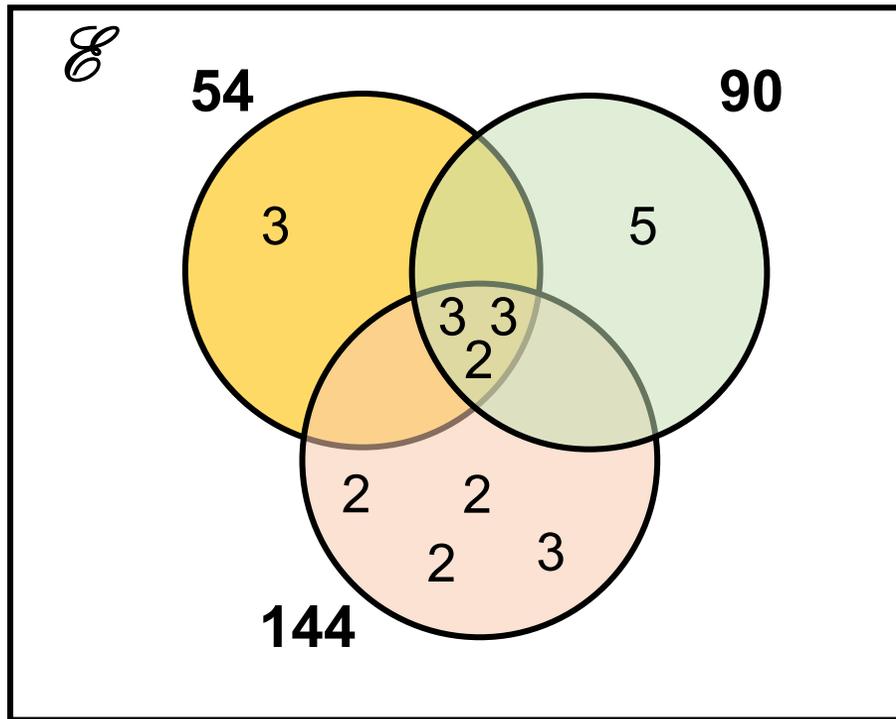d) 735 , 1617
   HCF = 147
   LCM = 8085

Example

Find the HCF and LCM of 54 , 90 and 144.

Solution

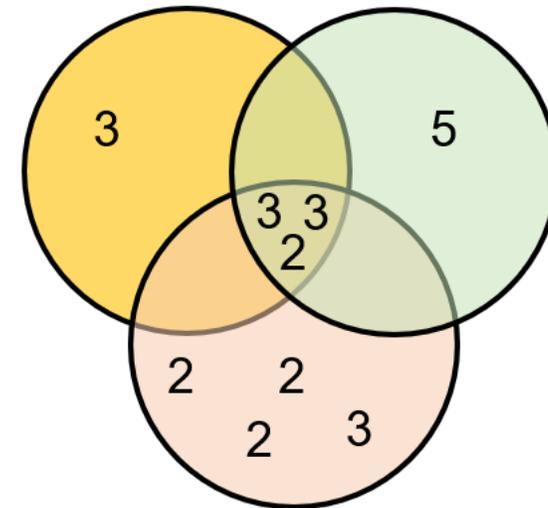$54 = 2 \times 3 \times 3 \times 3$

$90 = 2 \times 3 \times 3 \times 5$

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$HCF = 2 \times 3 \times 3 = \underline{\textbf{18}}$

$LCM = 2^4 \times 3^3 \times 5 = \underline{\textbf{2160}}$

Exercise

Find the HCF and LCM of:

a) 2 , 3 and 4

b) 6 , 8 and 10

c) 5 , 10 and 15

d) 1 , 4 and 16

Find the HCF and LCM of each pair of numbers:

a) 8 , 16 and 64

b) 12 , 80 and 96

c) 7 , 11 and 19

Find the HCF and LCM of each set of numbers:

a) 90 , 120 and 360

b) 39 , 91 and 273

Solutions

Find the HCF and LCM of:

a) 2 , 3 and 4

   HCF = 1, LCM = 12

b) 6 , 8 and 10

   HCF = 2, LCM = 120

c) 5 , 10 and 15

   HCF = 5, LCM = 30

d) 1 , 4 and 16

   HCF = 1, LCM = 16

Find the HCF and LCM of each pair of numbers:

a) 8 , 16 and 64

   HCF = 8, LCM = 64

b) 12 , 80 and 96

   HCF = 4, LCM = 480

c) 7 , 11 and 19

   HCF = 1, LCM = 1463

Find the HCF and LCM of each set of numbers:

a) 90 , 120 and 360

   HCF = 30
   LCM = 360

b) 39 , 91 and 273

   HCF = 13
   LCM = 273

Extension

Find the LCM and HCF of the numbers 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 , 10.

Solution

Find the LCM and HCF of the numbers 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 , 10.

LCM = 2 x 3 x 2 x 5 x 7 x 2 x 3 = **2520**

HCF = **1**

Written as a product of its prime factors

$240 = 2^4 \times 3 \times 5$

a) Write 220 as a product of its prime factors.

b) Work out the highest common factor of 240 and 220.

c) Work out the lowest common multiple of 240 and 220.

Solution

Written as a product of its prime factors

$240 = 2^4 \times 3 \times 5$

a) Write 220 as a product of its prime factors.

220

22 x 10

2 x 11 x 2 x 5   *2² x 5 x 11*

ℰ

240   **220**

2   2

2   2   **11**

3

5

b) Work out the highest common factor of 240 and 220.

2² x 5   *= 20*

c) Work out the lowest common multiple of 240 and 220.

2⁴ x 3 x 5 x 11   *= 2 640*
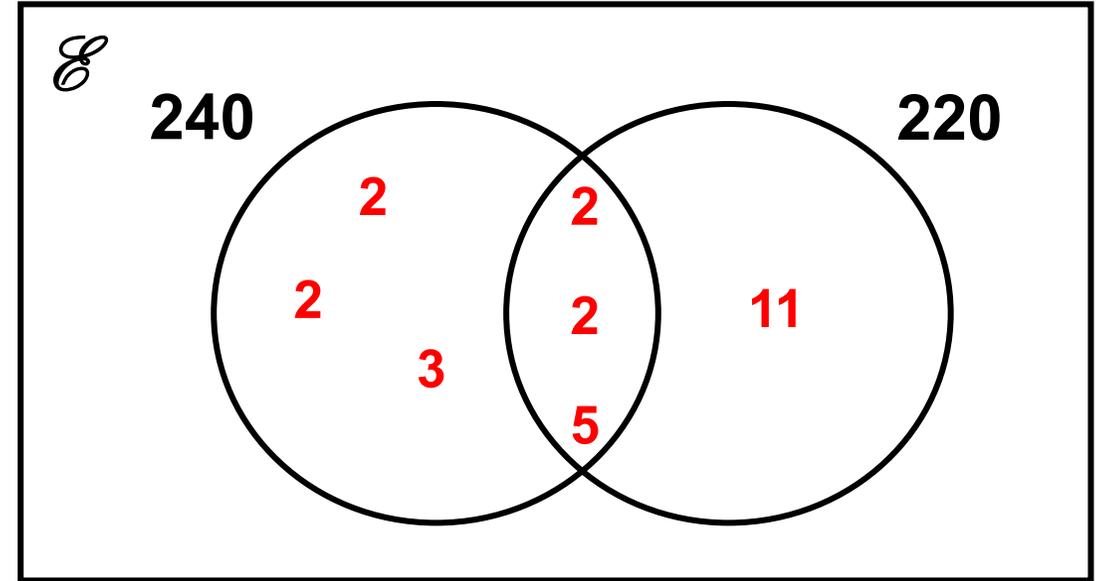
A and B are two numbers.

Written as a product of prime factors,.

$$A = 2^2 \times 3^2 \times p^2$$

$$B = 2^3 \times 3 \times p^3$$

a) The highest common factor of A and B is 1452

   Work out the value of p..

b) Work out the lowest common multiple of A and B.

A and B are two numbers.

Written as a product of prime factors,.

$$A = 2^2 \times 3^2 \times p^2$$

$$B = 2^3 \times 3 \times p^3$$

a) The highest common factor of A and B is 1452

   Work out the value of p..

b) Work out the lowest common multiple of A and B.

a) *HCF = 1452*

   $2^2 \times 3 \times p^2 = 1452$

   $12p^2 = 1452$

   $p^2 = 121$

   <u>*p = 11*</u>

b) *LCM = $2^3 \times 3^2 \times 11^3$*

   <u>*= 95 832*</u>

a) $x = 2^6 \times 3^2 \times 5^4$

Work out 12x.

Give your answer as a product of prime factors in index form.

b) $y = 2^3 \times 3^2 \times c^4$

Work out $5y^2$ as a product of prime factors in index form.

Give your answer in terms of c.

a) $x = 2^6 \times 3^2 \times 5^4$

Work out 12x.

Give your answer as a product of prime factors in index form.

$$12 = 2^2 \times 3$$

$$12x = 2^8 \times 3^3 \times 5^4$$

b) $y = 2^3 \times 3^2 \times c^4$

Work out $5y^2$ as a product of prime factors in index form.

Give your answer in terms of c.

$$5y^2 = 5 \times (2^3 \times 3^2 \times c^4)^2$$

$$= 5 \times (2^6 \times 3^4 \times c^8)$$

$$= 2^6 \times 3^4 \times 5 \times c^8$$