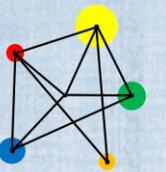


Indices



Calculate powers of numbers

Calculate square roots and cube roots

Understand and use rules of indices

Calculate simple negative and fractional indices

Vocabulary

Superscript

A number written above and to the right of another letter or number. For example, in 2^3 the '3' is the superscript.

Index

A synonym for power. The plural of index is indices.

Power

The power (or exponent) of a number says how many times to use the number in a multiplication. It is written as a small number to the right and above the base number.

Review of Powers and Roots

Evaluate:

1) 2^2

2) 5^2

3) 10^2

4) 9^2

5) 2^3

6) $\sqrt{25}$

Evaluate:

1) 13^2

2) 2^5

3) 7^3

4) 1^8

5) $\sqrt{225}$

6) $\sqrt[3]{27}$

Evaluate:

1) 2^{10}

2) $(-2)^3$

3) $\left(\frac{3}{5}\right)^3$

4) $\sqrt{1089}$

5) $\sqrt[3]{1331}$

6) $\sqrt[5]{243}$

Solutions

Evaluate:

1) $2^2 = 4$

2) $5^2 = 25$

3) $10^2 = 100$

4) $9^2 = 81$

5) $2^3 = 8$

6) $\sqrt{25} = 5$

Evaluate:

1) $13^2 = 169$

2) $2^5 = 32$

3) $7^3 = 343$

4) $1^8 = 1$

5) $\sqrt{225} = 15$

6) $\sqrt[3]{27} = 3$

Evaluate:

1) $2^{10} = 1024$

2) $(-2)^3 = -8$

3) $\left(\frac{3}{5}\right)^3 = \frac{27}{125}$

4) $\sqrt{1089} = 33$

5) $\sqrt[3]{1331} = 11$

6) $\sqrt[5]{243} = 3$

Extra practice

Evaluate:

1. 2^6

7. 2^{10}

13. 0.9^2

19. $(-3)^2$

2. 10^5

8. 11^3

14. 0.2^2

20. $(-2)^3$

3. 3^4

9. 0^6

15. 0.25^2

21. $(-1)^5$

4. 2^8

10. 5^4

16. $\left(\frac{1}{2}\right)^2$

22. $\left(-\frac{1}{3}\right)^2$

5. 1^9

11. 25^2

17. $\left(\frac{1}{10}\right)^2$

23. $2^2 \times 3^2$

6. 7^3

12. 15^2

18. $\left(\frac{2}{5}\right)^2$

24. $(2 \times 3)^2$

Solutions

Evaluate:

1. $2^6 = 64$

7. $2^{10} = 1024$

13. $0.9^2 = 0.81$

19. $(-3)^2 = 9$

2. $10^5 = 100\,000$

8. $11^3 = 1331$

14. $0.2^2 = 0.04$

20. $(-2)^3 = -8$

3. $3^4 = 81$

9. $0^6 = 0$

15. $0.25^2 = 0.0625$

21. $(-1)^5 = -1$

4. $2^8 = 256$

10. $5^4 = 625$

16. $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

22. $\left(-\frac{1}{3}\right)^2 = \frac{1}{9}$

5. $1^9 = 1$

11. $25^2 = 625$

17. $\left(\frac{1}{10}\right)^2 = \frac{1}{100}$

23. $2^2 \times 3^2 = 36$

6. $7^3 = 343$

12. $15^2 = 225$

18. $\left(\frac{2}{5}\right)^2 = \frac{4}{25}$

24. $(2 \times 3)^2 = 36$

Examples

Simplify:

a) $2^8 \times 2^5$

b) $2^8 \div 2^5$

c) $(2^8)^5$

Solutions

Simplify:

$$\begin{aligned} \text{a) } 2^8 \times 2^5 &= (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) \\ &= \underline{2^{13}} \end{aligned}$$

$$\begin{aligned} \text{b) } 2^8 \div 2^5 &= \frac{2 \times 2 \times 2 \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}} \\ &= \underline{2^3} \end{aligned}$$

$$\begin{aligned} \text{c) } (2^8)^5 &= 2^8 \times 2^8 \times 2^8 \times 2^8 \times 2^8 \\ &= \underline{2^{40}} \end{aligned}$$

Rules of Indices

$$x^a \times x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

Examples

a) $5^2 \times 5^4 = 5^?$

b) $4^8 \div 4^3 = 4^?$

c) $\frac{2^7}{2^3} = 2^?$

d) $(3^2)^3 = 3^?$

Solutions

a) $5^2 \times 5^4 = 5^6$

b) $4^8 \div 4^3 = 4^5$

c) $\frac{2^7}{2^3} = 2^4$

d) $(3^2)^3 = 3^6$

Write each expression as a single power

1. $3^4 \times 3^5 =$

2. $5^6 \div 5^2 =$

3. $\frac{6^8}{6^5} =$

4. $2^3 \times 2 =$

5. $4^6 \div 4^0 =$

6. $9 \times 3^2 =$

Write each expression as a single power

1. $2^2 \times 2^3 \times 2^4 =$

2. $\frac{4^2 \times 4^5}{4^3} =$

3. $7^6 \div 7^6 =$

4. $2^3 \div 2^4 =$

5. $(2^3)^2 =$

6. $(3 \times 3^2)^2 =$

$$2^6 = 4^3$$

Why are they equal?

Can you find another pair?

Write each expression as a single power

1. $3^4 \times 3^5 = 3^9$

2. $5^6 \div 5^2 = 5^4$

3. $\frac{6^8}{6^5} = 6^3$

4. $2^3 \times 2 = 2^4$

5. $4^6 \div 4^0 = 4^6$

6. $9 \times 3^2 = 3^4$

Write each expression as a single power

1. $2^2 \times 2^3 \times 2^4 = 2^9$

2. $\frac{4^2 \times 4^5}{4^3} = 4^4$

3. $7^6 \div 7^6 = 7^0 = 1$

4. $2^3 \div 2^4 = 2^{-1}$

5. $(2^3)^2 = 2^6$

6. $(3 \times 3^2)^2 = 3^6$

$$2^6 = 4^3$$

Why are they equal?

$$4 = 2^2$$

Can you find another pair?

$$2^4 = 4^2$$

Further Rules of Indices

$$x^0 = 1$$

$$x^{-a} = \frac{1}{x^a}$$

$$x^{\left(\frac{a}{b}\right)} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$$

Examples

a) $6^0 = ?$

b) $3^{-2} = ?$

c) $8^{\frac{2}{3}} = ?$

d) $\left(\frac{16}{25}\right)^{-\frac{3}{2}} = ?$

Solutions

$$\text{a) } 6^0 = \mathbf{1}$$

$$\text{b) } 3^{-2} = \frac{\mathbf{1}}{\mathbf{9}}$$

$$\text{c) } 8^{\frac{2}{3}} = \mathbf{4}$$

$$\text{d) } \left(\frac{16}{25}\right)^{-\frac{3}{2}} = \frac{\mathbf{125}}{\mathbf{64}}$$

Evaluate without a calculator:

a) 7^0

b) 2^{-1}

c) $9^{\frac{1}{2}}$

d) 5^{-2}

e) $16^{\frac{1}{4}}$

f) 1^{-1}

g) 2^{-5}

h) $27^{\frac{2}{3}}$

i) $25^{-\frac{1}{2}}$

j) $\left(\frac{5}{8}\right)^0$

k) $81^{\frac{3}{4}}$

l) $\left(\frac{4}{49}\right)^{\frac{1}{2}}$

m) $(-3)^{-2}$

n) $100^{\frac{3}{2}}$

o) 0.25^2

p) $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$

q) $\left(-\frac{1}{27}\right)^{\frac{2}{3}}$

r) $\left(\frac{32}{243}\right)^{\frac{4}{5}}$

s) $\left(\frac{4}{9}\right)^{-\frac{1}{2}}$

t) $\left(1\frac{11}{25}\right)^{-\frac{3}{2}}$

u) $3.375^{-\frac{1}{3}}$

v) $1024^{0.2}$

w) $\left(\frac{5}{7}\right)^{-2}$

x) $3^{\frac{1}{2}} \times 3^{\frac{1}{2}}$

Evaluate without a calculator:

$$\text{a) } 7^0 = \mathbf{1}$$

$$\text{b) } 2^{-1} = \frac{\mathbf{1}}{\mathbf{2}}$$

$$\text{c) } 9^{\frac{1}{2}} = \mathbf{3}$$

$$\text{d) } 5^{-2} = \frac{\mathbf{1}}{\mathbf{25}}$$

$$\text{e) } 16^{\frac{1}{4}} = \mathbf{2}$$

$$\text{f) } 1^{-1} = \mathbf{1}$$

$$\text{g) } 2^{-5} = \frac{\mathbf{1}}{\mathbf{32}}$$

$$\text{h) } 27^{\frac{2}{3}} = \mathbf{9}$$

$$\text{i) } 25^{-\frac{1}{2}} = \frac{\mathbf{1}}{\mathbf{5}}$$

$$\text{j) } \left(\frac{5}{8}\right)^0 = \mathbf{1}$$

$$\text{k) } 81^{\frac{3}{4}} = \mathbf{27}$$

$$\text{l) } \left(\frac{4}{49}\right)^{\frac{1}{2}} = \frac{\mathbf{2}}{\mathbf{7}}$$

$$\text{m) } (-3)^{-2} = \frac{\mathbf{1}}{\mathbf{9}}$$

$$\text{n) } 100^{\frac{3}{2}} = \mathbf{1\ 000}$$

$$\text{o) } 0.25^2 = \mathbf{0.0625}$$

$$\text{p) } \left(\frac{1}{4}\right)^{-\frac{1}{2}} = \mathbf{2}$$

$$\text{q) } \left(-\frac{1}{27}\right)^{\frac{2}{3}} = \frac{\mathbf{1}}{\mathbf{9}}$$

$$\text{r) } \left(\frac{32}{243}\right)^{\frac{4}{5}} = \frac{\mathbf{16}}{\mathbf{81}}$$

$$\text{s) } \left(\frac{4}{9}\right)^{-\frac{1}{2}} = \frac{\mathbf{3}}{\mathbf{2}}$$

$$\text{t) } \left(1\frac{11}{25}\right)^{-\frac{3}{2}} = \frac{\mathbf{125}}{\mathbf{216}}$$

$$\text{u) } 3.375^{-\frac{1}{3}} = \frac{\mathbf{2}}{\mathbf{3}}$$

$$\text{v) } 1024^{0.2} = \mathbf{4}$$

$$\text{w) } \left(\frac{5}{7}\right)^{-2} = \frac{\mathbf{49}}{\mathbf{25}}$$

$$\text{x) } 3^{\frac{1}{2}} \times 3^{\frac{1}{2}} = \mathbf{3}$$

Expressions with Indices

Simplify:

a) $2a^{\frac{1}{2}} \times 5a^2$

b) $8m^{-2} \div 2m^{-1}$

c) $\frac{2p^7 \times 4p^{-1}}{p^{-2}}$

d) $(2t^{-2})^{-3}$

Solutions

Simplify:

$$\text{a) } 2a^{\frac{1}{2}} \times 5a^2 = 10a^{\frac{5}{2}}$$

$$\text{b) } 8m^{-2} \div 2m^{-1} = 4m^{-1}$$

$$\text{c) } \frac{2p^7 \times 4p^{-1}}{3p^{-2}} = \frac{8}{3}p^8$$

$$\text{d) } (2t^{-2})^{-3} = \frac{1}{8}t^6$$

Exercise

Simplify each expression.

1. $p^{-1} \times 2p^{-1}$

2. $9w^{-3} \div 3w^5$

3. $\frac{6m^2}{3m^{-2}}$

4. $(2a^2)^3$

Simplify each expression.

1. $2xy^2 \times 2x^3y^{-3}$

2. $(3b^5)^2 \div b^6$

3. $\frac{3t^{-4} \times 2t^5}{4t^{-1}}$

4. $(2a^{-2})^{-4}$

Simplify:

1. $(125a^3)^{\frac{2}{3}}$

2. $\sqrt{64a^6b^{10}}$

Solutions

Simplify each expression.

1. $p^{-1} \times 2p^{-1}$ **$2p^{-2}$**

2. $9w^{-3} \div 3w^5$ **$3w^{-8}$**

3. $\frac{6m^2}{3m^{-2}}$ **$2m^4$**

4. $(2a^2)^3$ **$8a^6$**

Simplify each expression.

1. $2xy^2 \times 2x^3y^{-3}$
 $4x^4y^{-1}$

2. $(3b^5)^2 \div b^6$ **$9b^4$**

3. $\frac{3t^{-4} \times 2t^5}{4t^{-1}}$ **$\frac{3t^2}{2}$**

4. $(2a^{-2})^{-4}$ **$\frac{a^8}{16}$**

Simplify:

1. $(125a^3)^{\frac{2}{3}}$ **$25a^2$**

2. $\sqrt{64a^6b^{10}}$ **$8a^3b^5$**

1. Evaluate:

a) 11^2 b) 5^3 c) $\sqrt{196}$

d) $\sqrt[3]{125}$ e) $(-2)^2$ f) $\left(\frac{1}{2}\right)^3$

g) $\sqrt[5]{243}$ h) $\left(-\frac{2}{3}\right)^2$

2. Simplify:

a) $3^3 \times 3^4$ b) $5^{10} \div 5^5$ c) $\frac{6^7}{6^7}$

d) $\frac{7^4 \times 7^3}{7^5}$ e) $(2^4)^2$

3. Evaluate:

a) 9^0 b) 16^0 c) 8^{-1}

d) 3^{-2} e) $32^{\frac{2}{5}}$ f) $(8)^{-\frac{2}{3}}$

4. Simplify:

a) $y^2 \times y^4 \times y^6$ b) $\frac{x^2}{x^6}$

c) $(2b^5)^3$ d) $\frac{4w^2 \times 4w^{-1}}{w^{-2}}$

e) $\frac{2p^7 \times 4p^{-1}}{p^{-2}}$

Review Exercise

1. Evaluate:

$$\text{a) } 11^2 = 121$$

$$\text{b) } 5^3 = 125$$

$$\text{c) } \sqrt{196} = 14$$

$$\text{d) } \sqrt[3]{125} = 5$$

$$\text{e) } (-2)^2 = 4$$

$$\text{f) } \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$\text{g) } \sqrt[5]{243} = 3$$

$$\text{h) } \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$$

2. Simplify:

$$\text{a) } 3^3 \times 3^4 = 3^7$$

$$\text{b) } 5^{10} \div 5^5 = 5^5$$

$$\text{c) } \frac{6^7}{6^7} = 6^0 = 1$$

$$\text{d) } \frac{7^4 \times 7^3}{7^5} = 7^2$$

$$\text{e) } (2^4)^2 = 2^8$$

3. Evaluate:

$$\text{a) } 9^0 = 1 \quad \text{b) } 16^0 = 1 \quad \text{c) } 8^{-1} = \frac{1}{8}$$

$$\text{d) } 3^{-2} = \frac{1}{9} \quad \text{e) } 32^{\frac{2}{5}} = 4 \quad \text{f) } (8)^{-\frac{2}{3}} = \frac{1}{4}$$

4. Simplify:

$$\text{a) } y^2 \times y^4 \times y^6 = y^{12} \quad \text{b) } \frac{x^2}{x^6} = x^{-4}$$

$$\text{c) } (2b^5)^3 = 8b^{15} \quad \text{d) } \frac{4w^2 \times 4w^{-1}}{w^{-2}} = 16w^3$$

$$\text{e) } \frac{2p^7 \times 4p^{-1}}{p^{-2}} = 8p^8$$

Solutions

Extension

Solve the equations:

a) $x^2 = 100$

b) $2^x = 100$

Exam Style Question

- a) Write 9^2 as a power of 3
- b) Show that $3^4 \times 9^{-1}$ is a square number.
- c) Agnes has to work out the exact value of $27^{\frac{1}{3}}$

Agnes says,

$$\text{“ } \frac{1}{3} \text{ of } 27 \text{ is } 3 \text{ so } 27^{\frac{1}{3}} = 9 \text{”}$$

Is Agnes correct?

You must explain your answer.

Solution

a) Write 9^2 as a power of 3 $(3 \times 3) \times (3 \times 3) = 3^4$

b) Show that $3^4 \times 9^{-1}$ is a square number. $81 \times \frac{1}{9} = 9$

c) Agnes has to work out the exact value of $27^{\frac{1}{3}}$

Agnes says,

“ $\frac{1}{3}$ of 27 is 3 so $27^{\frac{1}{3}} = 9$ ”

Is Agnes correct?

You must explain your answer. ***Agnes is incorrect.***

$$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$$