# Special Sequences

Recognise square, triangular, cube , Fibonacci and prime numbers.

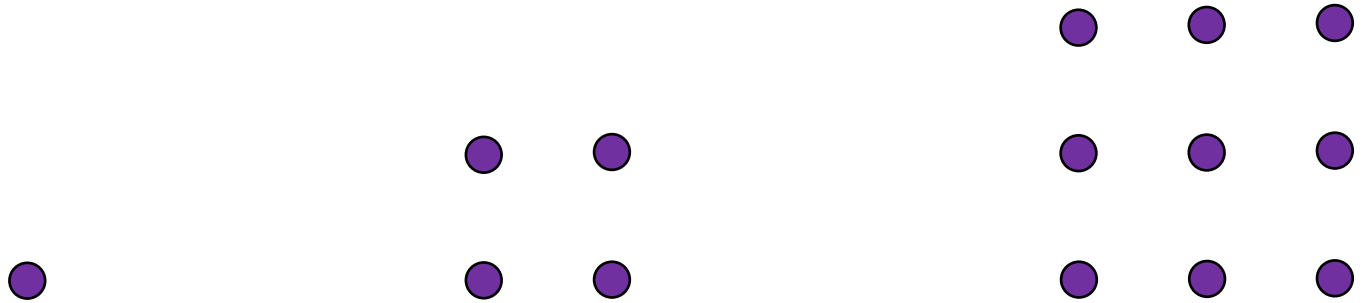
# Vocabulary

**Term**

A number in a sequence.

# Square Numbers

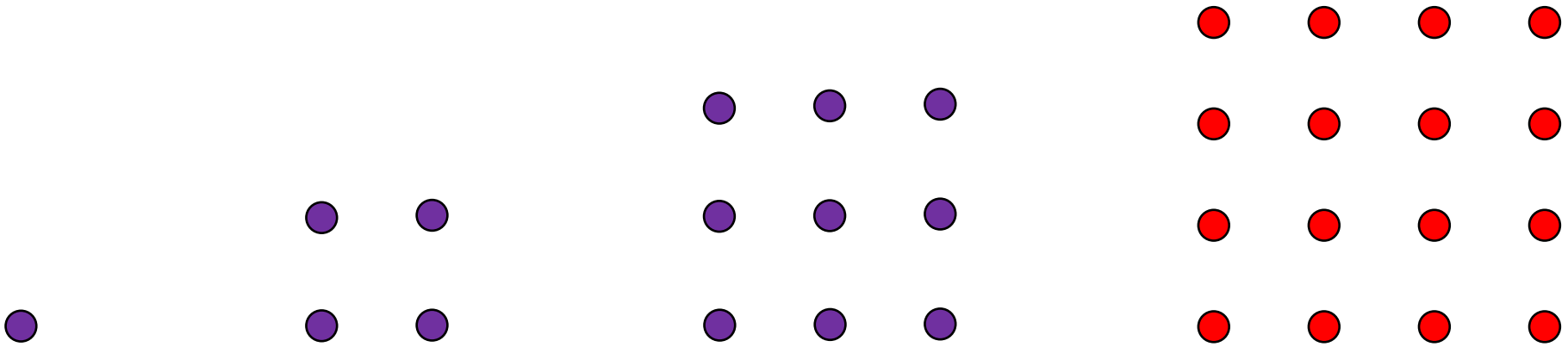
How many dots are there in the next pattern in the sequence?



How many dots are there in the 10th pattern in the sequence?

# Solutions

How many dots are there in the next pattern in the sequence?



**There are 16 dots in the next pattern.**

How many dots are there in the 10th pattern in the sequence?

**There are 100 dots in the 10th pattern.**

# Square Numbers

The n’th square number can be found using the n’th term expression $n^2$.

| Position in sequence (n) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| n’th square number ($n^2$) | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 |

# Example

The n'th square number can be found using the n'th term expression $n^2$.

a) Use the formula to find the 19th square number.

b) Is 1156 a square number?

**Challenge**

Is 647,242,918,035,492 a square number?

# Solution

| n | $n^2$ |
|---|---|
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |
| 5 | 25 |
| 6 | 36 |
| 7 | 49 |
| 8 | 64 |
| 9 | 81 |
| 10 | 100 |

| n | $n^2$ |
|---|---|
| 11 | 121 |
| 12 | 144 |
| 13 | 169 |
| 14 | 196 |
| 15 | 225 |
| 16 | 256 |
| 17 | 289 |
| 18 | 324 |
| 19 | 361 |
| 20 | 400 |

a) Use the formula to find the 19th square number. **$19^2 = 361$**

b) Is 1156 a square number?

**Yes. $1156 = 34^2$**

**Challenge**

**647,242,918,035,492 is not a square number.**

**Square numbers do not end with a 2**

# Exercise

1. Find: a) $13^2$ b) $30^2$

2. What is the $100^{th}$ square number?

3. 625 is the square of which number?

4. Is 1008 a square number?

5. 5929 is the square of which number?

6. How many square numbers are there between 100 and 200 inclusive?

7. 2025 was a square number year. When will the next one be?

**Challenge**

$14 = 3^2 + 2^2 + 1^2$

Can every number greater than 7 be expressed as the sum of 3 square numbers?

(You may use $0^2 = 0$
i.e. $10 = 3^2 + 1^2 + 0^2$)

# Solutions

1. Find: a) $13^2$ **169** b) $30^2$ **900**

2. What is the 100th square number? **10 000**

3. 625 is the square of which number? **25**

4. Is 1008 a square number? **No. Square numbers do not end with '8'.**

5. 5929 is the square of which number? **77**

6. How many square numbers are there between 100 and 200 inclusive? **5**

7. 2025 was a square number year. When will the next one be? **2116**

**Challenge**

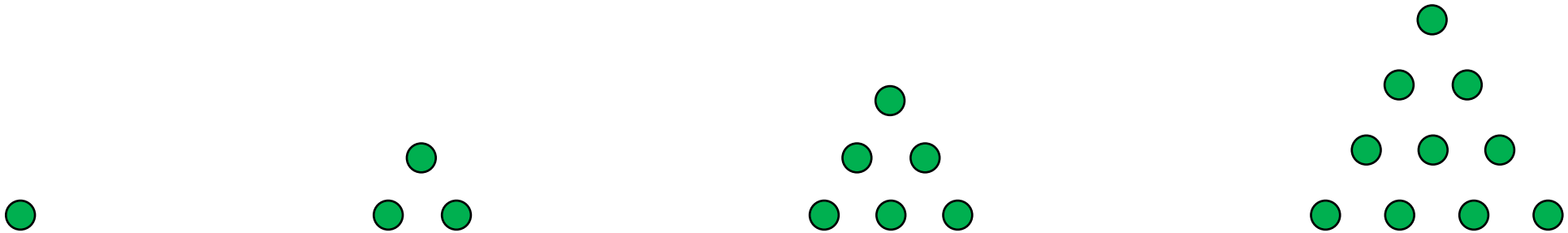
$14 = 3^2 + 2^2 + 1^2$

Can every number greater than 7 be expressed as the sum of 3 square numbers? **No.**

(You may use $0^2 = 0$
i.e. $10 = 3^2 + 1^2 + 0^2$)

# Triangular Numbers

Here is a set of **triangular** numbers.



**Pattern 1**

**Pattern 2**

**Pattern 3**

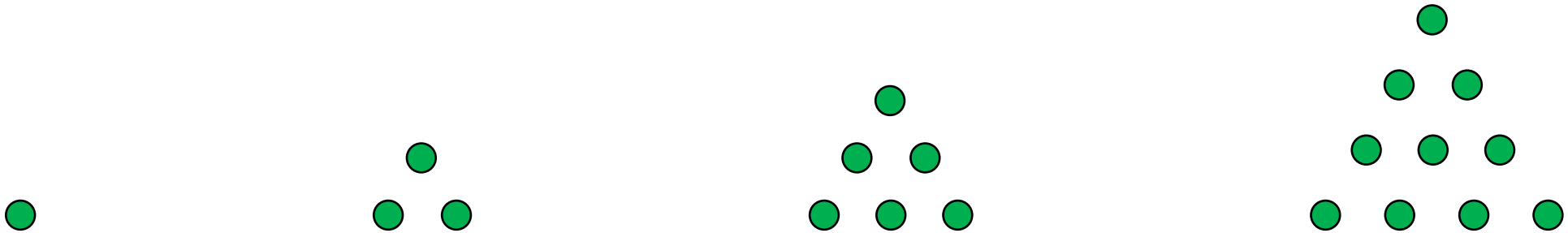
**Pattern 4**

How many dots are there in the next pattern in the sequence?

How many dots are there in the 10th pattern in the sequence?

# Solutions

Here is a pattern of **triangular** numbers.



**Pattern 1**

**Pattern 2**

**Pattern 3**

**Pattern 4**

How many dots are there in the next pattern in the sequence? **15 dots**

How many dots are there in the 10th pattern in the sequence? **55 dots**

# Triangular Numbers

The n’th triangular number can be found using the n’th term expression $\frac{n(n+1)}{2}$

| Position in sequence (n) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| n’th triangular number | 1 | 3 | 6 | 10 | 15 | 21 | 28 | 36 | 45 | 55 |

# Example

The n'th triangular number can be found using the n'th term expression $\frac{n(n+1)}{2}$

a) Use the formula to find the 99th triangular number.

b) Is 276 a triangular number.

# Example

The n'th triangular number can be found using the n'th term expression $\frac{n(n+1)}{2}$

a) Use the formula to find the 99th triangular number.

$$\frac{\mathbf{99 \times 100}}{\mathbf{2}} = \mathbf{4950}$$

b) Is 276 a triangular number. **Yes**

$1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210$

$231, 253, \mathbf{276}$

# Exercise

1. What is the twentieth triangular number?
2. Is 100 a triangular number?
3. Find two triangular numbers with a sum of 100.
4. Find two triangular numbers with a difference of 100.
5. Are any triangular numbers also square numbers?
6. When is the next ‘triangular year’?
7. Are any triangular numbers also prime numbers?

**Challenge**

Investigate what happens when you add consecutive pairs of triangular numbers?

# Solutions

1. What is the twentieth triangular number? **210**
2. Is 100 a triangular number? **No**
3. Find two triangular numbers with a sum of 100. **45 and 55**
4. Find two triangular numbers with a difference of 100. **4950 and 5950**
5. Are any triangular numbers also square numbers?

   **Yes. 1 , 36 , 1225…..**
   **There are infinitely many.**
6. When is the next ‘triangular year’? **2080**
7. Are any triangular numbers also prime numbers?

   **Yes. 3 is the only triangular number which is also prime.**

**Challenge**

Investigate what happens when you add consecutive pairs of triangular numbers?

**The results are square numbers.**

# Fibonacci Numbers

Here are the first ten Fibonacci numbers.

1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55

Can you see how the sequence is constructed?

What is the $20^{th}$ number in the sequence?

# Fibonacci Numbers

Here are the first ten Fibonacci numbers.

1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55

Can you see how the sequence is constructed?

**To obtain the next number, we add the previous two numbers.**

What is the $20^{th}$ number in the sequence?

**6765**

# Key Facts

In a Fibonacci sequence, the next term is found by adding the previous two terms.

The most famous Fibonacci sequence begins: 1 , 1 , …

1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55 , 89 , 144 , 233 , 377 , 610 , 987 , 1597 , 2584 , …

# Exercise

Find the next terms in each Fibonacci sequence

a) 1 , 2 , 3 , ___ , ___

b) 1 , 3 , 4 , ___ , ___

c) 10 , 10 , 20 , ___ , ___

d) - 1 , 1 , ___ , ___ , ___

e) $\frac{1}{4}$ , $\frac{1}{2}$ , ___ , ___ , ___

Find the missing terms in each Fibonacci sequence

a) 1 , ___, 5 , ___ , ___

b) ___ , 5 , 7 , ___ , ___

c) ___ , ___ , ___ , 12 , 20

d) 1 , ___ , ___ , 13 , ___

e) 5 , ___ , ___ , ___ , 46

What is the fifth term of this Fibonacci sequence?

a , b , ……

# Solutions

Find the next terms in each Fibonacci sequence

a) 1 , 2 , 3 , **5** , **8**

b) 1 , 3 , 4 , **7** , **11**

c) 10 , 10 , 20 , **30** , **50**

d) - 1 , 1 , **0** , **1** , **1**

e) $\frac{1}{4}$ , $\frac{1}{2}$ , $\frac{3}{4}$ , $1\frac{1}{4}$ , **2**

Find the missing terms in each Fibonacci sequence

a) 1 , **4** , 5 , **9** , **14**

b) **2** , 5 , 7 , **12** , **19**

c) **4** , **4** , **8** , 12 , 20

d) 1 , **6** , **7** , 13 , **20**

e) 5 , **12** , **17** , **29** , 46

What is the fifth term of this Fibonacci sequence?

a , b , a + b , a + 2b ,

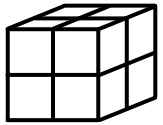
**2a + 3b**

# Cube Numbers

Here is a pattern of **cube** numbers.

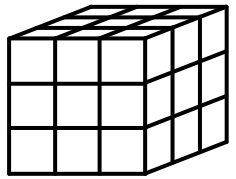
**Pattern 1**

*1 cube*



**Pattern 2**

*8 cubes*



**Pattern 3**

*27 cubes*

How many cubes are in the next pattern in the sequence?

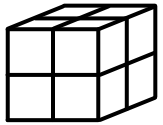
How many cubes are there in the 20th pattern in the sequence?

# Solution

Here is a pattern of **cube** numbers.

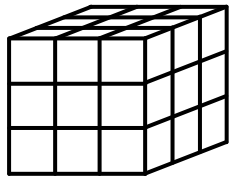
**Pattern 1**

*1 cube*



**Pattern 2**

*8 cubes*



**Pattern 3**

*27 cubes*

How many cubes are in the next pattern in the sequence?

**$4^3$ = 64 cubes**

How many cubes are there in the 20th pattern in the sequence?

**$20^3$ = 8 000 cubes**

# Cube Numbers

The n’th cube number can be found using the n’th term expression $n^3$.

| Position in sequence (n) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| n’th cube number ($n^3$) | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 | 1000 |

# Exercise

1. What is the 100th cube number?

2. How many cube numbers are there between 1000 and 2000 inclusive?

3. 15 625 is the cube of which number

4. Is 1 billion a cube number?

5. Find three cube numbers that are also square numbers?

6. $3^3 + 4^3 + 5^3 = ?^3$

**Challenge**

$1^3 = 1$

$1^3 + 2^3 = 9$

$1^3 + 2^3 + 3^3 = 36$

$1^3 + 2^3 + 3^3 + 4^3 =$ ……

Can you find a pattern?

# Solutions

1. What is the 100th cube number?
   **1 000 000**

2. How many cube numbers are there between 1000 and 2000 inclusive?
   **3**

3. 15 625 is the cube of which number
   **25**

4. Is 1 billion a cube number?
   **Yes $1000^3$**

5. Find three cube numbers that are also square numbers?
   **1 , 64 , 729…..**
   **There are infinitely many.**

6. $3^3 + 4^3 + 5^3 = \mathbf{6^3}$

**Challenge**

$1^3 = 1$

$1^3 + 2^3 = 9$

$1^3 + 2^3 + 3^3 = 36$

$1^3 + 2^3 + 3^3 + 4^3 =$ ……

Can you find a pattern?

**The sums of cube numbers are square numbers.**

# Extension

$5^2 + 12^2 = 13^2$

Can you find another three numbers that satisfy: $a^2 + b^2 = c^2$

How many groups of three numbers can you find?

# Exam Style Question

a) Here is a list of numbers.

21 22 23 24 25 26 27 28 29

(i) From the numbers in the list, write down a square number.

(ii) From the numbers in the list, write down a cube number.

b) Show that 280 can be written as the sum of a square number and a cube number.

# Solution

a) Here is a list of numbers.

21 22 23 24 25 26 27 28 29

(i) From the numbers in the list, write down a square number. *25*

(ii) From the numbers in the list, write down a cube number. *27*

b) Show that 280 can be written as the sum of a square number and a cube number.

*$280 = 8^2 + 6^3$*