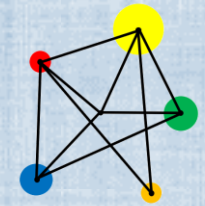
Expanding and Factorising



Expand a single bracket.

Factorise an expression by finding a common factor.

Vocabulary

Expression
A collection of letters, numbers and mathematical symbols
i.e. $2x + y - 2$.

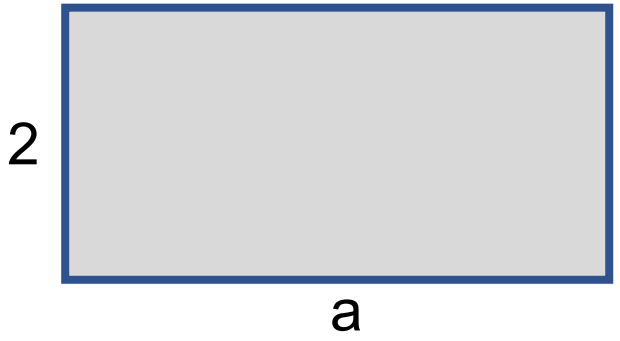
Expand
Multiply two or more expressions or brackets together.

Common Factor
A number or expression that is common to all parts of a larger expression.

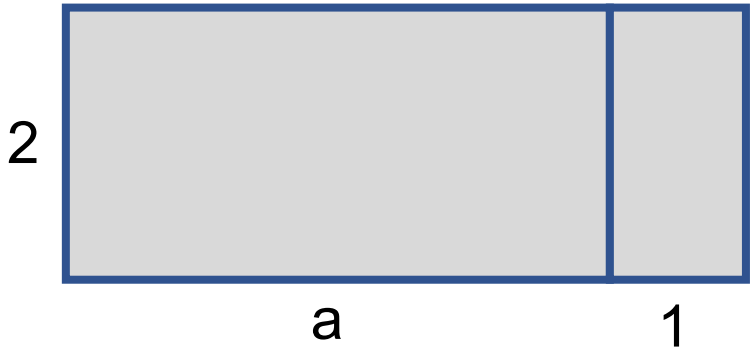
Factorise
Find a ‘common factor’ in an expression and use it to write the expression with brackets.

Question

What is the area of this rectangle?



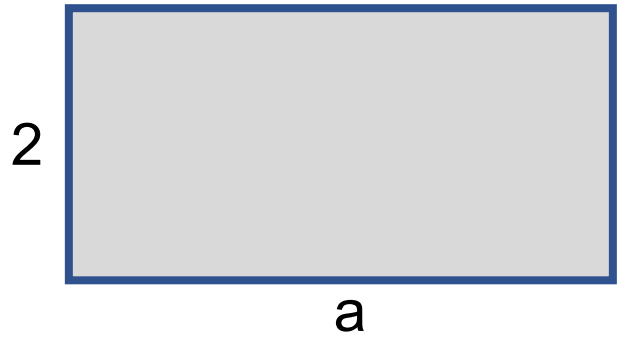
What is the area of the larger rectangle?



Solutions

Area of rectangle

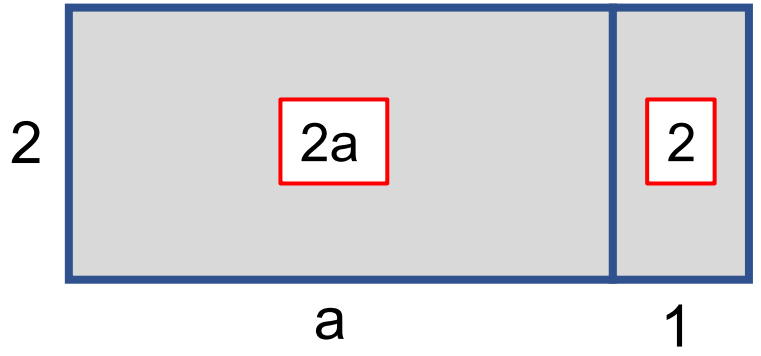
= 2a



Area of the larger rectangle

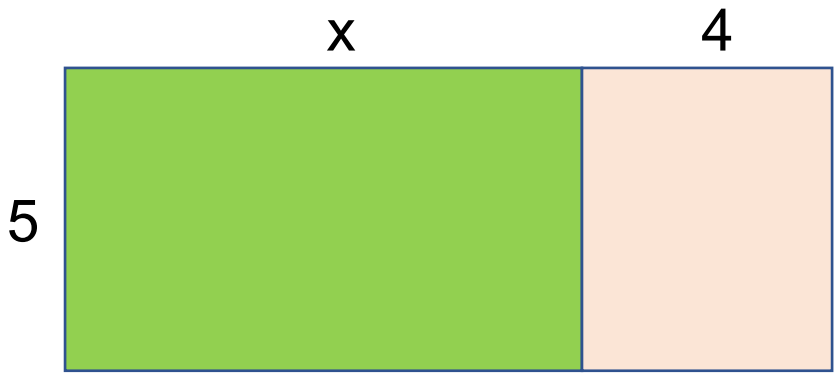
= 2(a + 1)

= 2a + 2



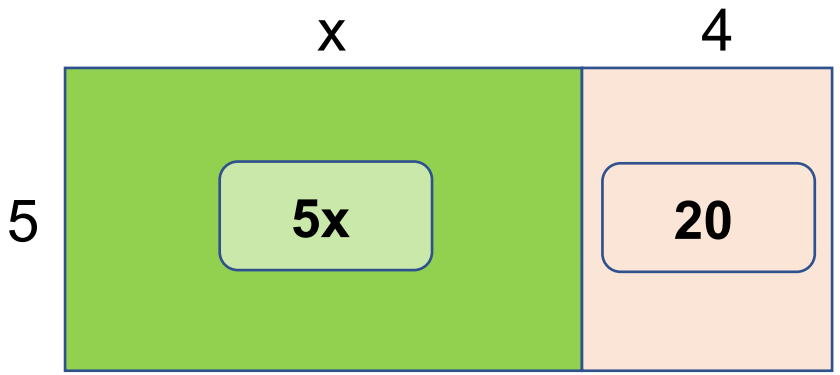
Example

Write down an expression for the area of this rectangle made of two smaller rectangles.



Solutions

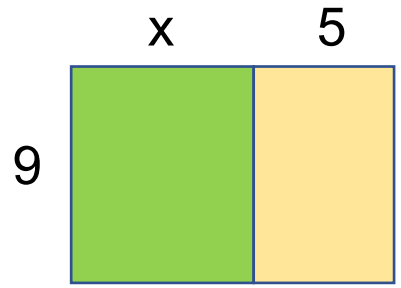
Write down an expression for the area of this rectangle made of two smaller rectangles.

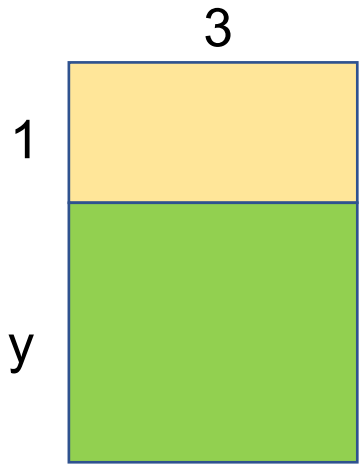


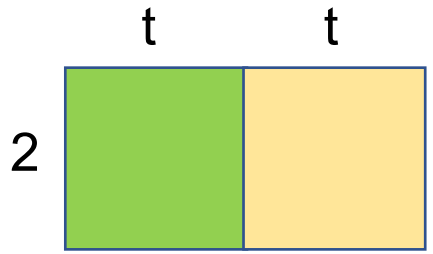
Area = $5(x + 4)$

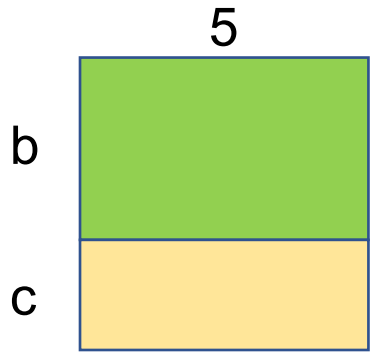
$= \underline{5x + 20}$

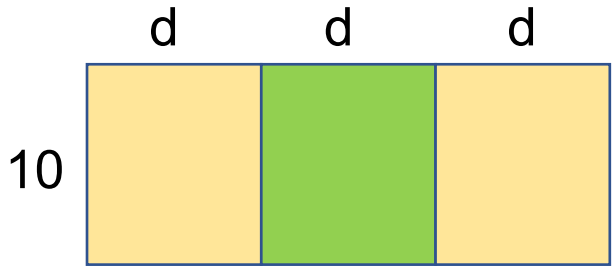
Write down an expression for the area of each shape.

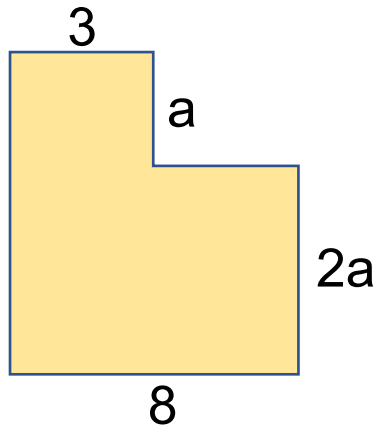




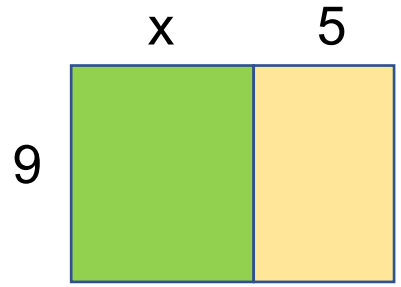




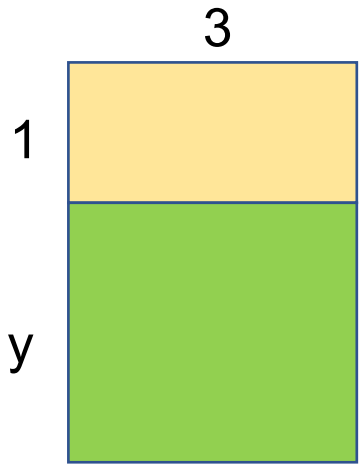




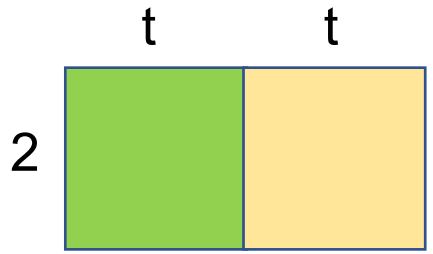
Write down an expression for the area of each shape.



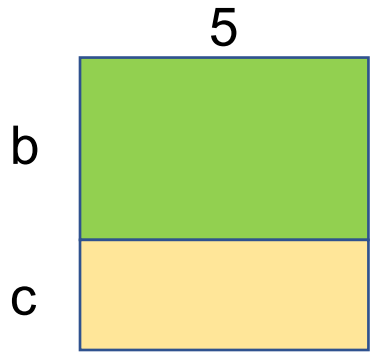
$9(x + 5) = 9x + 45$



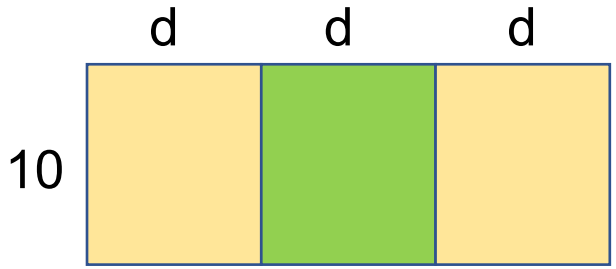
$3(1 + y) = 3 + 3y$



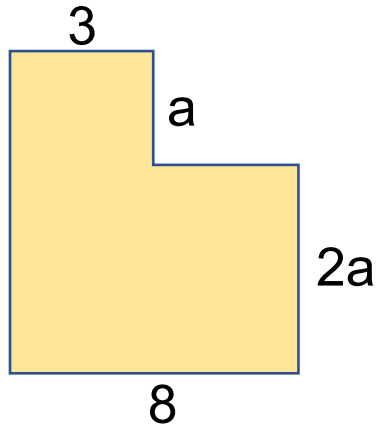
$2(t + t) = 4t$



$5(b + c) = 5b + 5c$



$10(d + d + d) = 30d$



$3a + 16a = 19a$

Examples

a) Expand $4(x + 3)$

b) Expand $3(x – 1)$

Solutions

a) Expand $4(x + 3)$

$= 4x + 12$

b) Expand $3(x - 1)$

$= 3x - 3$

Exercise

Expand.

1. $3(x + 2)$
2. $4(a + 1)$
3. $2(b + 6)$
4. $5(t + 10)$
5. $6(m + 5)$
6. $-8(p + 3)$

Expand.

1. $5(x - 2)$
2. $6(p - 1)$
3. $3(1 - t)$
4. $-5(t - 2)$
5. $-3(n - 5)$

Expand.

1. $-4(2 - s)$
2. $6(4p - 6)$
3. $-5(-4 - 3v)$
4. $\frac{2}{3}(6w - 1)$

Solutions

Expand.

1. $3(x + 2)$ **$= 3x + 6$**
2. $4(a + 1)$ **$= 4a + 4$**
3. $2(b + 6)$ **$= 2b + 12$**
4. $5(t + 10)$ **$= 5t + 50$**
5. $6(m + 5)$ **$= 6m + 30$**
6. $-8(p + 3)$ **$= -8p - 24$**

Expand.

1. $5(x - 2)$ **$= 5x - 10$**
2. $6(p - 1)$ **$= 6p - 6$**
3. $3(1 - t)$ **$= 3 - 3t$**
4. $-5(t - 2)$ **$= -5t + 10$**
5. $-3(n - 5)$ **$= -3n + 15$**

Expand.

1. $-4(2 - s)$ **$= -8 + 4s$**
2. $6(4p - 6)$ **$= 24p - 36$**
3. $-5(-4 - 3v)$ **$= 20 + 15v$**
4. $\frac{2}{3}(6w - 1)$ **$= 4w - \frac{2}{3}$**

Examples

a) Expand $p(p + 7)$

b) Expand $4m(2m – 3t)$

Solutions

a) Expand $p(p + 7)$

$= p^2 + 7p$

b) Expand $4m(2m – 3t)$

$= 8m^2 – 12mt$

Exercise

Expand.

1. $x(x + 8)$
2. $y(y - 4)$
3. $m(2 - m)$
4. $x(x + y)$
5. $- 2t(t - 3)$
6. $k^3(k^2 - 1)$

Expand.

1. $2x(x + 6)$
2. $3d(3d - 1)$
3. $2g(3 – 2g)$
4. $2p(p – p^2)$
5. $3m(n + m)$
6. $w(2w – w^2)$

Expand.

1. $2x(2x^2 + 3)$
2. $6y(5y^2 - 1)$
3. $2b^2(b^3 - b)$
4. $2c^2(1 – 2c^2)$
5. $3xy(x + y)$

Solutions

Expand.

1. $x(x + 8)$ **$= x^2 + 8x$**
2. $y(y - 4)$ **$= y^2 - 4y$**
3. $m(2 - m)$ **$= 2m - m^2$**
4. $x(x + y)$ **$= x^2 + xy$**
5. $-2t(t - 3)$ **$= -2t^2 + 6t$**
6. $k^3(k^2 - 1)$ **$= k^5 - k^3$**

Expand.

1. $2x(x + 6)$ **$= 2x^2 + 12x$**
2. $3d(3d - 1)$ **$= 9d^2 - 3d$**
3. $2g(3 - 2g)$ **$= 6g - 4g^2$**
4. $2p(p - p^2)$ **$= 2p^2 - 2p^3$**
5. $3m(n + m)$ **$= 3mn + 3m^2$**
6. $w(2w - w^2)$ **$= 2w^2 - w^3$**

Expand.

1. $2x(2x^2 + 3)$ **$= 4x^3 + 6x$**
2. $6y(5y^2 - 1)$ **$= 30y^3 - 6y$**
3. $2b^2(b^3 - b)$ **$= 2b^5 - 2b^3$**
4. $2c^2(1 - 2c^2)$ **$= 2c^2 - 4c^4$**
5. $3xy(x + y)$ **$= 3x^2y + 3xy^2$**

Example

Expand and simplify:

a) $2(w + 6) - 4$

b) $4 - 5(x - 1)$

Solutions

Expand and simplify:

a) $2(w + 6) - 4$

$= 2w + 12 - 4$

$= \mathbf{2w + 8}$

b) $4 - 5(x - 1)$

$= 4 - 5x + 5$

$= \mathbf{9 - 5x}$

Exercise

Expand and simplify:

1. $3(x + 2) + 7$
2. $5(y + 3) - 20$
3. $2(m - 4) + 2m$
4. $10(5 – 2p) + 5p$
5. $6 + 2(a + 1)$
6. $8 – 8(a – 1)$

Expand and simplify:

1. $2(3b + 7) + b$
2. $7(3a - 7) + 50$
3. $2(x + 7) + 7(x + 2)$
4. $6(2m - 1) - (2m + 2)$
5. $3(1 - y) - 3(y - 1)$

Expand and simplify:

1. $d(d + 7) + d$
2. $t(t – t^2) + t^3$

Exercise

Expand and simplify:

1. $3(x + 2) + 7$
   $3x + 13$
2. $5(y + 3) - 20$
   $5y - 5$
3. $2(m - 4) + 2m$
   $4m - 8$
4. $10(5 – 2p) + 5p$
   $50 - 15p$
5. $6 + 2(a + 1)$
   $8 + 2a$
6. $8 – 8(a – 1)$
   $16 – 8a$

Expand and simplify:

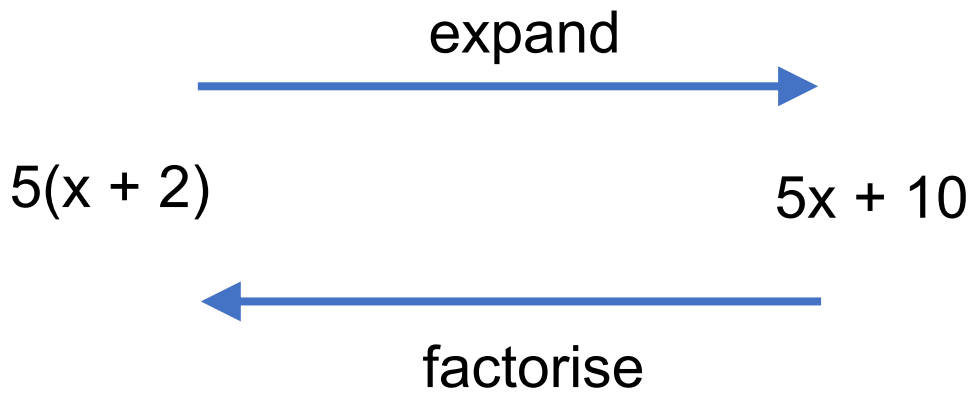
1. $2(3b + 7) + b$
   $7b + 14$
2. $7(3a - 7) + 50$
   $21a + 1$
3. $2(x + 7) + 7(x + 2)$
   $9x + 28$
4. $6(2m - 1) - (2m + 2)$
   $10m - 8$
5. $3(1 - y) + 3(y - 1)$ **0**

Expand and simplify:

1. $d(d + 7) + d$
   $d^2 + 8d$
2. $t(t – t^2) + t^3$
   t^2

Key Fact

The opposite process of expanding is called **factorising**.



Examples

a) Factorise $3x + 6$

b) Factorise *completely* $4x^2 – 2x^3 + 8x$

Solutions

a) Factorise $3x + 6$

$= 3(x + 2)$

b) Factorise *completely* $4x^2 – 2x^3 + 8x$

$= 2x(2x – x^2 + 4x)$

Factorising Exercise

Factorise.

1. $3x + 6$
2. $2y + 20$
3. $10p - 2$
4. $21 – 3q$
5. $- 5v - 75$
6. $- 11r + 121$

Factorise completely.

1. $a^2 + 4a$
2. $u^2 - u$
3. $2c^2 – 4c$
4. $2k – k^2$
5. $3y^3 + 6y$
6. $3y^3 + 6y^2$

Factorise completely..

1. $3xy + 9x$
2. $xy^2 – x^2y$
3. $4mn – 8m^2n^2$

Factorising Exercise

Factorise.

1. $3x + 6$ **$= 3(x + 2)$**
2. $2y + 20$ **$= 2(y + 10)$**
3. $10p - 2$ **$= 2(5p – 1)$**
4. $21 – 3q$ **$= 3(7 – q)$**
5. $- 5v - 75$ **$= - 5(v + 15)$**
6. $- 11r + 121$ **$= - 11(r – 11)$**

Factorise completely.

1. $a^2 + 4a$ **$= a(a + 4)$**
2. $u^2 - u$ **$= u(u – 1)$**
3. $2c^2 – 4c$ **$= 2c(c – 2)$**
4. $2k – k^2$ **$= k(2 – k)$**
5. $3y^3 + 6y$ **$= 3y(y^2 + 2)$**
6. $3y^3 + 6y^2$ **$= 3y^2(y + 2)$**

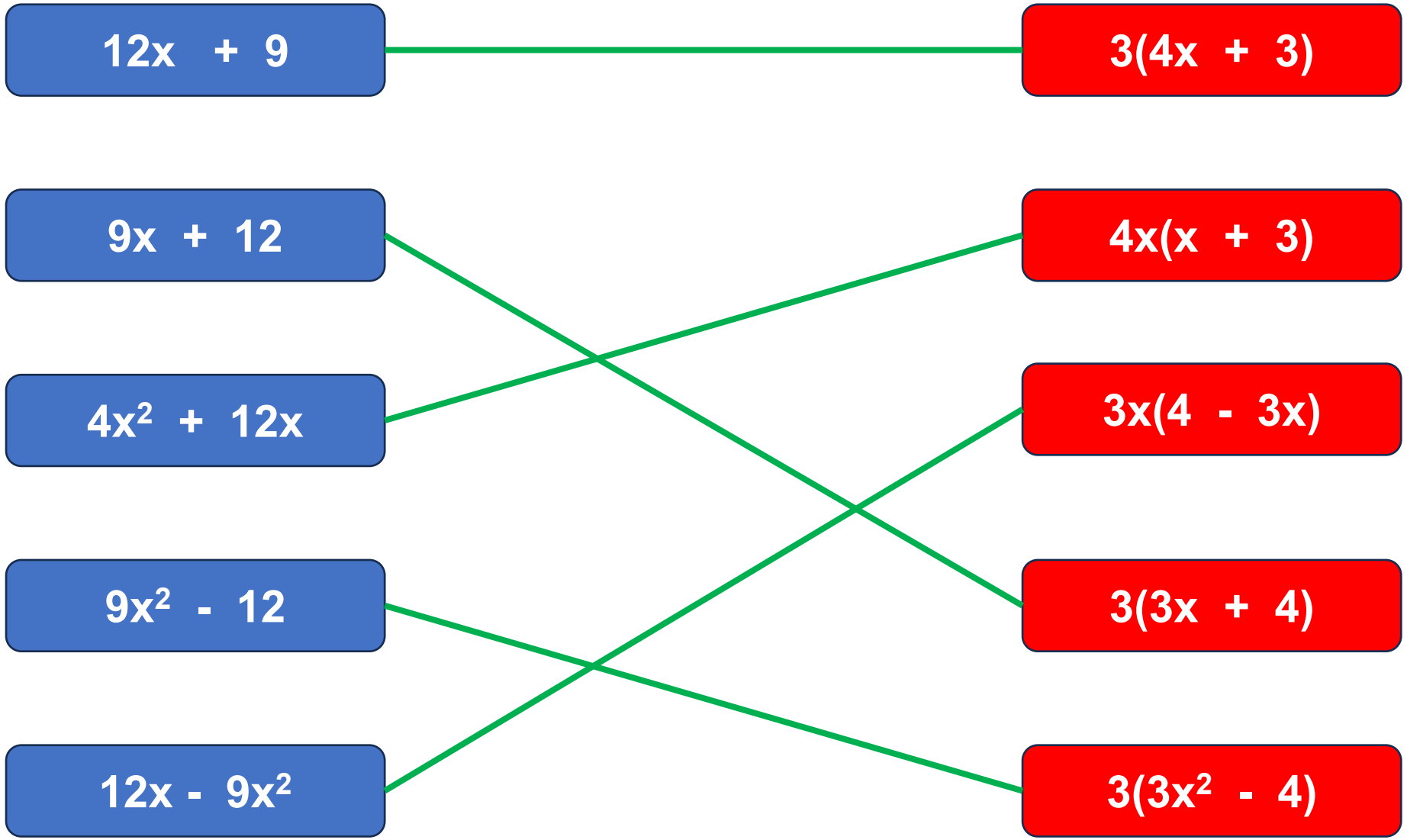
Factorise completely..

1. $3xy + 9x$ **$= 3x(y + 3)$**
2. $xy^2 – x^2y$ **$= xy(y – x)$**
3. $4mn – 8m^2n^2$ **$= 4mn(1 – 2mn)$**

Match up the equivalent expressions.

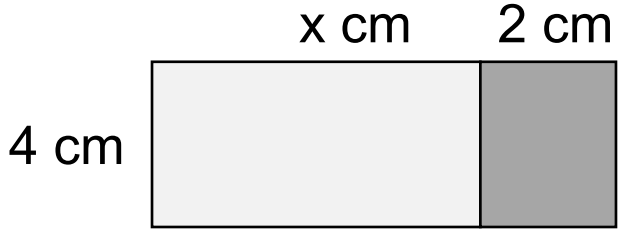
$12x + 9$	$3(4x + 3)$
$9x + 12$	$4x(x + 3)$
$4x^2 + 12x$	$3x(4 - 3x)$
$9x^2 - 12$	$3(3x + 4)$
$12x - 9x^2$	$3(3x^2 - 4)$

Match up the equivalent expressions.



Review

1. Write two different expressions for the total area of this rectangle.



2. Expand

a) $2(x + 7)$

b) $3(4x - 7)$

c) $8(1 - 2x)$

d) $\frac{1}{2}(x - \frac{1}{2})$

3. Expand

a) $x(x + 3)$

b) $m(m - 1)$

c) $y(3 - 2y)$

d) $2p(3p - 5)$

e) $t^3(t - t^3)$

4. Expand and simplify

a) $2(x + 1) + 8$

b) $4(2m - 3) - 5$

c) $6(1 + 2t) - 12t$

d) $3(x - 7) + 2(x - 1)$

5. Factorise

a) $2x + 6$

b) $8x - 4$

c) $x^2 + x$

d) $x^2 - 6x$

e) $2x^2 - 10x$

6. Factorise completely

a) $2x + 6$

b) $8x - 4$

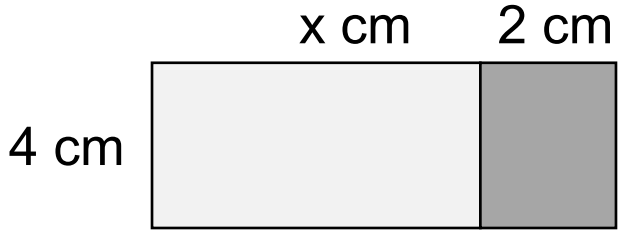
c) $x^2 + x$

d) $x^2 - 6x$

e) $2x^2 - 10x$

Solutions

1. Write two different expressions for the total area of this rectangle.



4x + 8 = 4(x + 2)

2. Expand

a) $2(x+7)$ **2x + 14**

b) $3(4x-7)$ **12x - 21**

c) $8(1-2x)$ **8 – 16x**

d) $\frac{1}{2}(x-\frac{1}{2})$ $\frac{1}{2}x-\frac{1}{4}$

3. Expand

a) $x(x+3)$ **x^2 + 3x**

b) $m(m-1)$ **m^2 - m**

c) $y(3-2y)$ **3y – $2y^2$**

d) $2p(3p-5)$ **$6p^2$ – 10p**

e) $t^3(t-t^3)$ **t^4 – t^6**

4. Expand and simplify

a) $2(x+1)+8$ **2x + 10**

b) $4(2m-3)-5$ **8m - 12**

c) $6(1+2t)-12t$ **6**

d) $3(x-7)+2(x-1)$

5x - 23

5. Factorise

a) $2x+6$ **2(x + 3)**

b) $8x-4$ **4(2x - 1)**

c) x^2+x **x(x + 1)**

d) x^2-6x **x(x - 6)**

e) $2x^2-10x$ **2x(x - 5)**

6. Factorise completely

a) $2x+6$ **2(3x + 5)**

b) $8x-4$ **4(3y - 2)**

c) x^2+x **2x(2x - 1)**

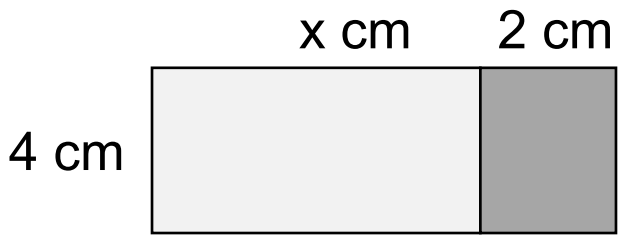
d) x^2-6x **3x(y - 3)**

e) $2x^2-10x$

2xy(xy^2 - 5)

Solutions

1. Write two different expressions for the total area of this rectangle.



4x + 8 = 4(x + 2)

2. Expand

1. $2(x+7)$ **2x + 14**
2. $3(4x-7)$ **12x - 21**
3. $8(1-2x)$ **8 – 16x**
4. $\frac{1}{2}(x-\frac{1}{2})$ $\frac{1}{2}x-\frac{1}{4}$

3. Expand

a) $x(x+3)$

b) $m(m-1)$

c) $y(3-2y)$

d) $2p(3p-5)$

e) $t^3(t-t^3)$

4. Expand and simplify

1. $2(x+1)+8$
2. $4(2m-3)-5$
3. $6(1+2t)-12t$
4. $3(x-7)+2(x-1)$

5. Factorise

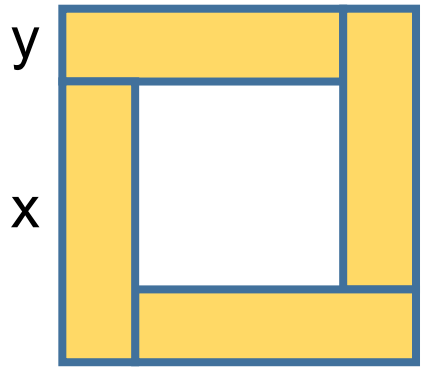
1. $2x+6$
2. $8x-4$
3. x^2+x
4. x^2-6x
5. $2x^2-10x$

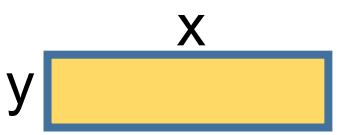
6. Factorise completely

1. $6x+10$
2. $12y-8$
3. $4x^2-2x$
4. $3xy-9x$
5. $2x^2y^3-10xy$

Extension

The shape below consists of four identical rectangles.





Each rectangle has a length of y and a width of x.

Show that the total perimeter of the shape is 8x.

Exam Style Question

Below is an identity.

$$a(5x - 4) \equiv 25x + 2b$$

Work out the values of a and b.

Solution

$a(5x - 4) \equiv 25x + 2b$

Work out the values of a and b.

Expand the bracket:

$5ax - 4a \equiv 25x + 2b$

$5a = 25$ so $a = 5$

$2b = -20$ so $b = -10$