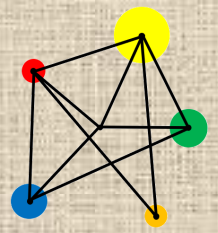
Inequalities



Use all four inequality signs.

Illustrate inequalities on a number line.

Solve linear inequalities.

Shade simple linear inequalities on co-ordinate axes.

Vocabulary

Inequality
A statement that one quantity is bigger (or smaller) than another.

Integer
A whole number.

Starter

Which of the signs $<$, $=$, $>$ should go between each pairs of numbers?

a) $3 \quad \_\_\_\_ \quad 1$

b) $-1 \quad \_\_\_\_ \quad 1$

c) $3^2 \quad \_\_\_\_ \quad 2^3$

d) $(-1)^2 \quad \_\_\_\_ \quad 1$

e) $-7 \quad \_\_\_\_ \quad -3$

f) $0.2 \quad \_\_\_\_ \quad \frac{1}{4}$

g) $\frac{3}{5} \quad \_\_\_\_ \quad \frac{2}{3}$

h) $\frac{1}{3} \quad \_\_\_\_ \quad \frac{1}{4}$

i) $0.3^2 \quad \_\_\_\_ \quad \frac{1}{10}$

j) $\frac{11}{10} \quad \_\_\_\_ \quad \frac{10}{9}$

Solutions

Which of the signs < , = , > should go between each pairs of numbers?

a) $3 \;\_>\_\; 1$

b) $-1 \;\_<\_\; 1$

c) $3^2 \;\_>\_\; 2^3$

d) $(-1)^2 \;\_=\_\; 1$

e) $-7 \;\_<\_\; -3$

f) $0.2 \;\_<\_\; \frac{1}{4}$

g) $\frac{3}{5} \;\_<\_\; \frac{2}{3}$

h) $\frac{1}{3} \;\_>\_\; \frac{1}{4}$

i) $0.3^2 \;\_<\_\; \frac{1}{10}$

j) $\frac{11}{10} \;\_<\_\; \frac{10}{9}$

Key Facts

$<$ means '**less than**'

$>$ means '**greater than**'

$\leq$ means '**less than or equal to**'

$\geq$ means '**greater than or equal to**'

Examples

Which of these statements are true?

a) $2 < 3$

b) $2 \leq 3$

c) $3 > 2$

d) $3 \geq 2$

e) $3 \leq 3$

e) $-3 \leq -3$

Solutions

Which of these statements are true?

a) $2 < 3$ **True**

b) $2 \leq 3$ **True**

c) $3 > 2$ **True**

d) $3 \geq 2$ **True**

e) $3 \leq 3$ **True**

e) $-3 \leq -3$ **True**

True or False

True or false?

1. $5 > 2$
2. $6 \geq 4$
3. $6 \geq 6$
4. $-1 < -2$
5. $-4 \leq -7$
6. $-3 \leq -1$

True or false?

1. $0.6 > 0.21$
2. $-5 < -7$
3. $\frac{1}{5} < \frac{1}{6}$
4. $0.8 \geq \frac{1}{8}$
5. $3^2 \leq 2^3$

True or false?

1. $-\frac{1}{3} > \frac{1}{2}$
2. $0.1^2 < 0.1$
3. $\frac{5}{6} < \frac{6}{5}$
4. $3\frac{1}{3} > \sqrt{10}$

True or False

True or false?

1. $5 > 2$ **True**
2. $6 \geq 4$ **True**
3. $6 \geq 6$ **True**
4. $-1 < -2$ **False**
5. $-4 \leq -7$ **False**
6. $-3 \leq -1$ **True**

True or false?

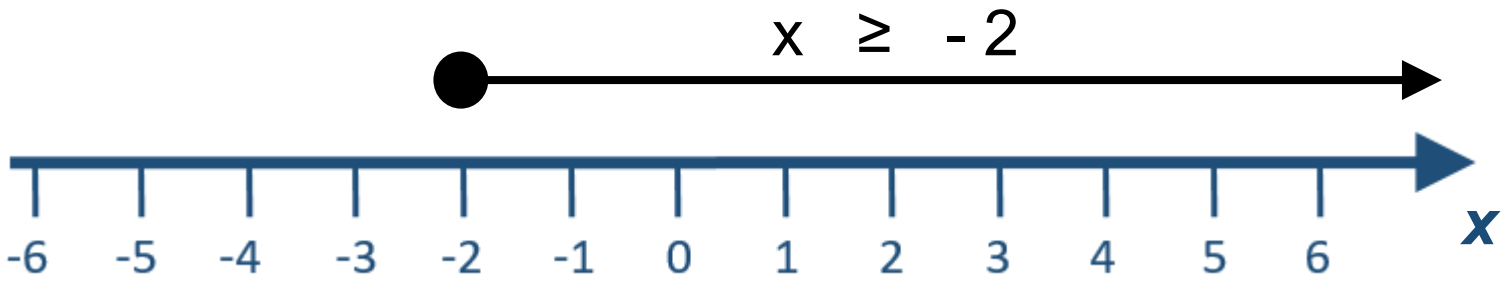
1. $0.6 > 0.21$ **True**
2. $-5 < -7$ **False**
3. $\frac{1}{5} < \frac{1}{6}$ **False**
4. $0.8 \geq \frac{1}{8}$ **True**
5. $3^2 \leq 2^3$ **False**

True or false?

1. $-\frac{1}{3} > \frac{1}{2}$ **False**
2. $0.1^2 < 0.1$ **True**
3. $\frac{5}{6} < \frac{6}{5}$ **True**
4. $3\frac{1}{3} > \sqrt{10}$ **True**

Key Fact

Inequalities can be represented on a number line:



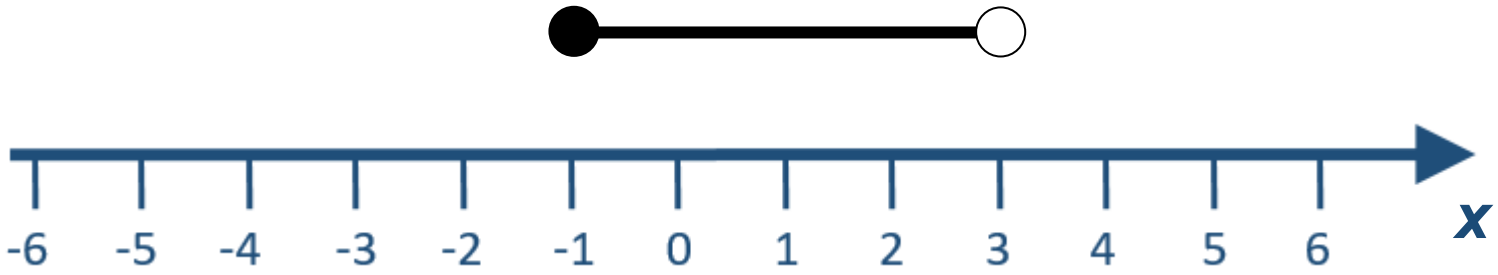
A solid circle is used to show that - 2 satisfies the inequality.

Example

Represent the following inequality on a number line: $-1 \leq x < 3$

Solution

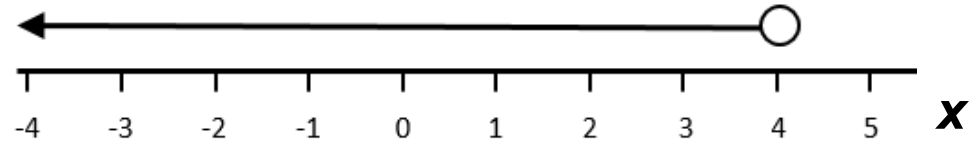
Represent the following inequality on a number line: $-1 \leq x < 3$

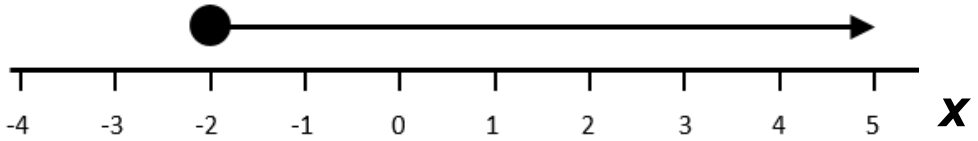


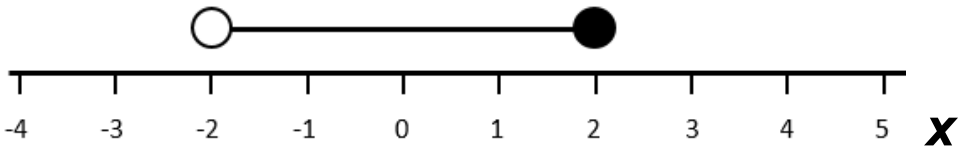
We use an empty circle to show that 3 is **not** included in the inequality.

You try …

Write down the inequalities shown by these number lines?





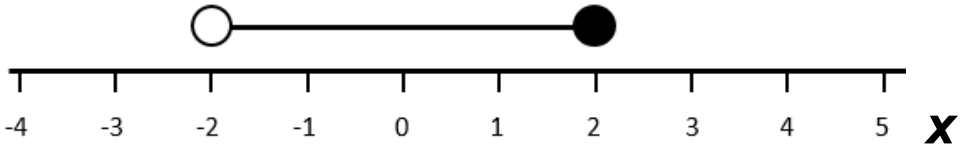


Solutions

Write down the inequalities shown by these number lines?

$x < 4$

$x \geq -2$



$-2 < x \leq 2$

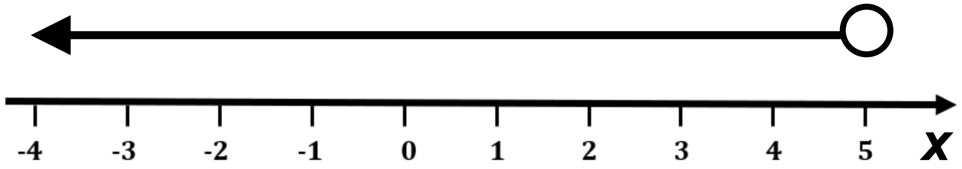
Exercise

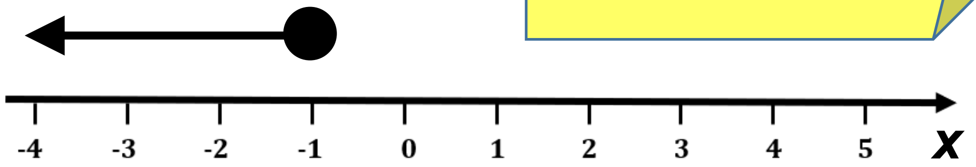
Challenge

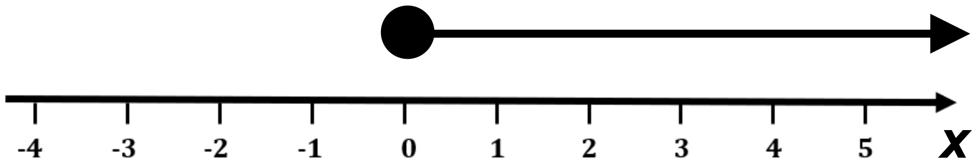
Which is larger:

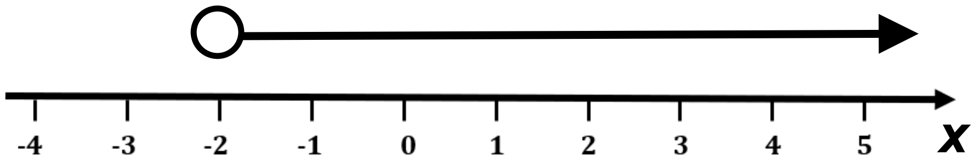
x^2 or x

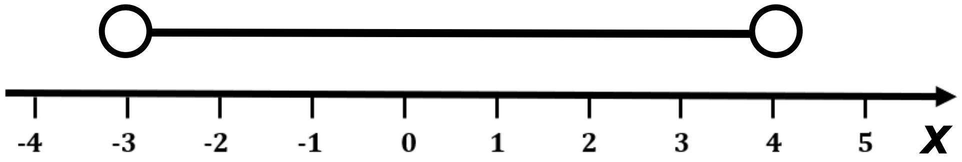
Write down the inequalities shown on each number line:

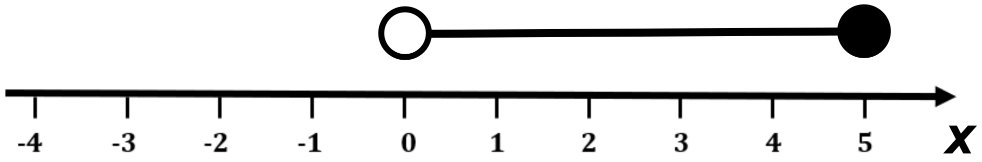






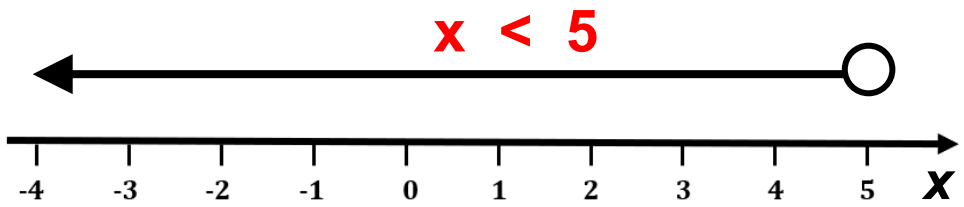


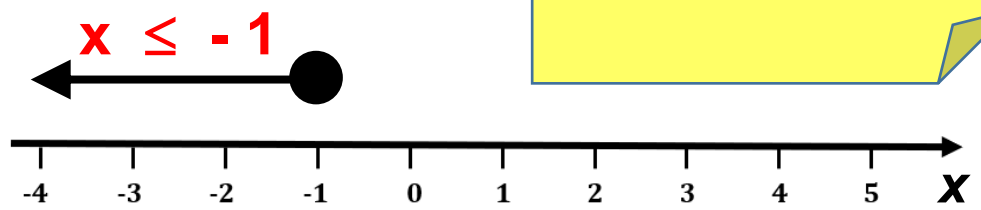


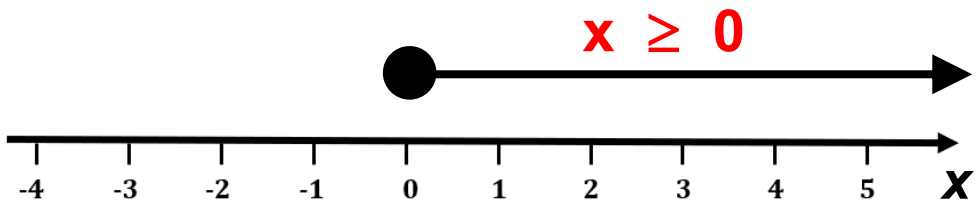


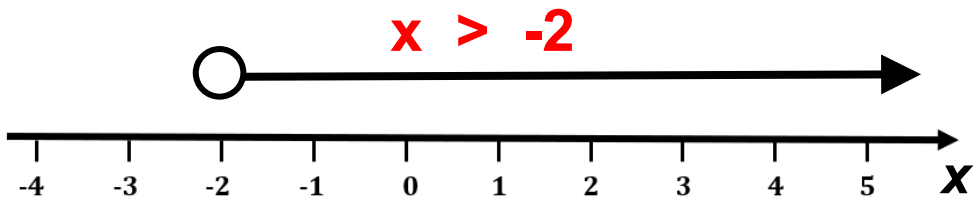
Solutions

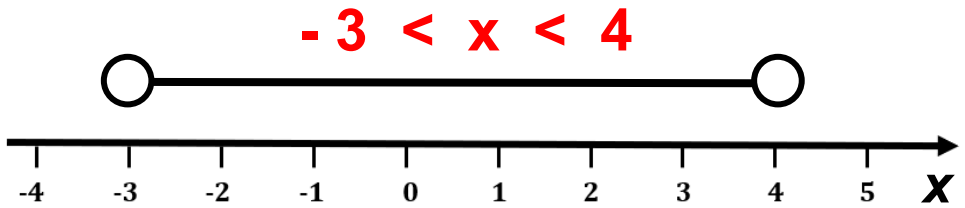
Write down the inequalities shown on each number line:

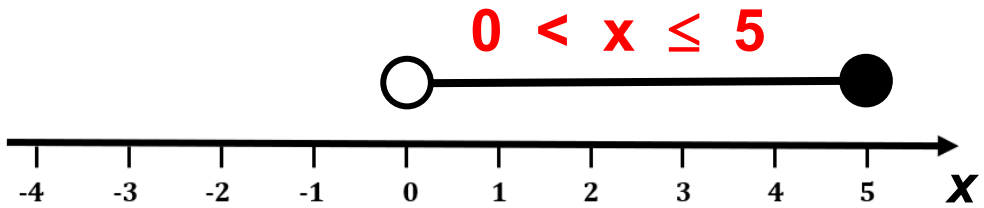












Which is larger:

x^2 or x

x when

0 < x < 1

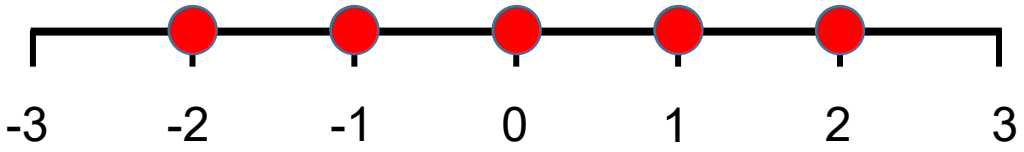
Example

n is an integer such that $-2 \leq n < 3$

Write down all the possible values of n.

Solution

n is an integer such that $-2 \leq n < 3$



The possible values of ‘n’ are : **- 2 , - 1 , 0 , 1 , 2**

Exercise

Write down the integers which satisfy each inequality:

(a) $-2 \leq n \leq 2$

(b) $3 \leq n < 6$

(c) $-1 < n < 1$

(d) $-4 \leq n + 3 < 0$

(e) $2 < 2n \leq 10$

(f) $-6 < 3n - 3 \leq 3$

Challenge

n is an integer that satisfies both inequalities below:

$-7 < n - 6 \leq 1$

$13 < 2n + 1 \leq 21$

What is the value of n?

Exercise

Write down the integers which satisfy each inequality:

(a) $-2 \leq n \leq 2$

$-2, -1, 0, 1, 2$

(b) $3 \leq n < 6$

$3, 4, 5$

(c) $-1 < n < 1$

0

(d) $-4 \leq n + 3 < 0$

$-7, -6, -5, -4$

(e) $2 < 2n \leq 10$

$2, 3, 4, 5$

(f) $-6 < 3n - 3 \leq 3$

$0, 1, 2$

Challenge

n is an integer that satisfies both inequalities below:

$-7 < n - 6 \leq 1$

$13 < 2n + 1 \leq 21$

What is the value of n?

$n = 7$

Solving Linear Inequalities Example

Solve the inequalities.

a) $b + 10 < 7$

b) $8x \leq 48$

c) $2t + 1 > 37$

d) $\frac{w - 6}{3} \geq 7$

Solutions

Solve the inequalities.

a) $b + 10 < 7$

$b < -3$

b) $8x \leq 48$

$x \leq 6$

c) $2t + 1 > 37$

$t > 18$

d) $\frac{w - 6}{3} \geq 7$

$w \geq 27$

Exercise

Solve these linear inequalities

1. $t + 1 > 5$
2. $b - 5 < 12$
3. $k + 19 \leq 41$
4. $2a \geq 32$
5. $10q < 420$
6. $\frac{w}{2} \leq -7$

Solve these linear inequalities

1. $p - 2 < 0$
2. $d + 10 \leq 5$
3. $12 - q > 8$
4. $31 \geq 2f$
5. $2x > x$

Solve these linear inequalities

1. $5t + 3 > 10$
2. $\frac{m - 11}{11} \geq 11$

Solve these non-linear inequalities

1. $x^2 < 16$
2. $x^3 > 8$

Solutions

Solve these linear inequalities

1. $t + 1 > 5$ **$t > 4$**
2. $b - 5 < 12$ **$b < 17$**
3. $k + 19 \leq 41$ **$k \leq 22$**
4. $2a \geq 32$ **$a \geq 16$**
5. $10q < 420$ **$q < 42$**
6. $\frac{w}{2} \leq -7$ **$w \leq -14$**

Solve these linear inequalities

1. $p - 2 < 0$ **$p < 2$**
2. $d + 10 \leq 5$ **$d \leq -5$**
3. $12 - q > 8$ **$q < 4$**
4. $31 \geq 2f$ **$f \leq \frac{31}{2}$**
5. $2x > x$ **$x > 0$**

Solve these linear inequalities

1. $5t + 3 > 10$ **$t > \frac{7}{5}$**
2. $\frac{m - 11}{11} \geq 11$ **$m \geq 132$**

Solve these non-linear inequalities

1. $x^2 < 16$ **$-4 < x < -4$**
2. $x^3 > 8$ **$x > 2$**

Example

Solve the inequalities.

a) $\frac{2x+7}{3} \leq 11$

b) $20 - 3x > 2x + 5$

Solutions

Solve the inequalities.

a) $\frac{2x+7}{3} \leq 11$

x 3 x 3

$2x+7 \leq 33$

- 7 - 7

$2x \leq 26$

÷ 2 ÷ 2

$x \leq 13$

b) $20-3x > 2x+5$

+ 3x + 3x

$20 > 5x+5$

- 5 - 5

$15 > 5x$

÷ 5 ÷ 5

$3 > x$

Exercise

Solve these linear inequalities

1. $3p + 5 \leq 17$
2. $64 > 16 + 6x$
3. $12 - k < 6$
4. $3m - 7 > m + 3$
5. $20 - 4t \leq 15 + t$
6. $2(b + 3) \geq 30$

Solve these linear inequalities

1. $57 < 3(y - 2)$
2. $4(r + 3) \geq 18$
3. $6w - 2 \leq 2(w + 5)$
4. $5(a - 1) \geq 2(a + 5)$
5. $4(2 - h) < 1 - 2h$

Solve:

$$\frac{1}{3}(x + 5) \geq \frac{1}{2}(2x - 6)$$

Solutions

Solve these linear inequalities

1. $3p + 5 \leq 17$ **$p \leq 4$**
2. $64 > 16 + 6x$ **$x < 8$**
3. $12 - k < 6$ **$k > 6$**
4. $3m - 7 > m + 3$ **$m > 5$**
5. $20 - 4t \leq 15 + t$ **$t \geq 1$**
6. $2(b + 3) \geq 30$ **$b \geq 12$**

Solve these linear inequalities

1. $57 < 3(y - 2)$ **$y > 21$**
2. $4(r + 3) \geq 18$ **$r \geq 1.5$**
3. $6w - 2 \leq 2(w + 5)$ **$w \leq 3$**
4. $5(a - 1) \geq 2(a + 5)$ **$a \geq 5$**
5. $4(2 - h) < 1 - 2h$ **$h > 3.5$**

Solve:

$$\frac{1}{3}(x + 5) \geq \frac{1}{2}(2x - 6)$$

$x \leq 7$

Further exercise

Solve these linear inequalities

1. $\frac{3m+5}{4} < -4$
2. $\frac{3(d+5)}{4} < \frac{4(d-2)}{5}$
3. $9 < 2x + 1 \leq 20$

Solve these linear inequalities

1. $\frac{1}{4} < \frac{x}{3} < \frac{1}{2}$
2. $\frac{2}{h} \leq 0.4$
3. $5 - \frac{6}{t} \leq -1$

Solve these non-linear inequalities

1. $x^2 < 25$
2. $x^2 \geq 9$
3. $x^3 \leq 8$

Solutions

Solve these linear inequalities

1. $\frac{3m+5}{4} < -4$

   m < - 7

2. $\frac{3(d+5)}{4} < \frac{4(d-2)}{5}$

   d > 107

3. $9 < 2x + 1 \leq 20$

   4 < x ≤ 9.5

Solve these linear inequalities

1. $\frac{1}{4} < \frac{x}{3} < \frac{1}{2}$

   0.75 < x < 1.5

2. $\frac{2}{h} \leq 0.4$

   h ≥ 5 or h ≤ 0

3. $5 - \frac{6}{t} \leq -1$

   t ≤ 1

Solve these non-linear inequalities

1. $x^2 < 25$

   - 5 < x ≤ 5

2. $x^2 \geq 9$

   x ≤ - 3 or x ≥ 3

3. $x^3 \leq 8$

   x ≤ 2

Extension

Which of the following statements is true?

$0.9 < 1$

$0.99999999999999 < 1$

$0.\dot{9} < 1$

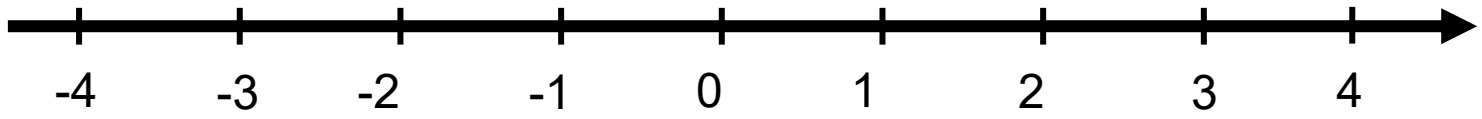
Exam Style Question

a) Solve $12n > 28 - 2n$

b) n is an integer.
$-3 < n \leq 3$.

Write down all the possible values of n

c) On the number line below, show the set of values for which $-8 \leq 3n+1 < 2$



Solution

a) Solve $12n > 28 - 2n$

$14n > 28$

$n > 2$

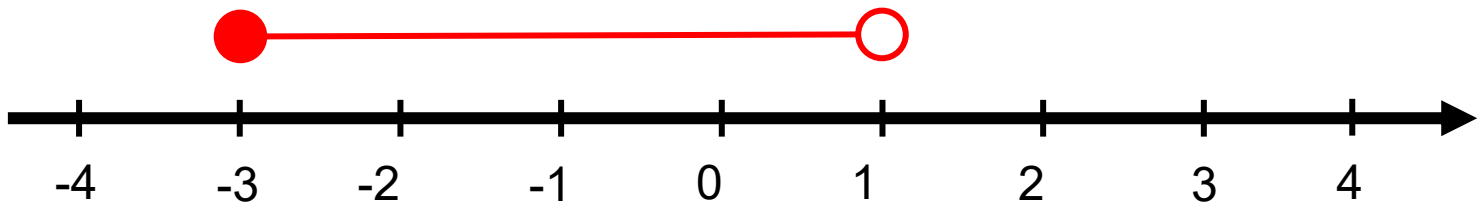
b) n is an integer.

$-3 < n \leq 3$.

Write down all the possible values of n

-2 , -1 , 0 , 1 , 2 , 3

c) On the number line below, show the set of values for which $-8 \leq 3n+1 < 2$



Exam Style Question

Here are two inequalities.

$$-1 \leq x < 4$$

$$8 \leq x + y \leq 12$$

x and y are integers.

Work out the **greatest** possible value of y - x.

Solution

Here are two inequalities.

$$-1 \leq x < 4$$

$$8 \leq x + y \leq 12$$

x and y are integers.

Work out the **greatest** possible value of y - x.

x = -1 , 0 , 1 , 2 , 3

We need the smallest possible value of x so x = -1

$$8 \leq 3 + y \leq 12$$

$$5 \leq y \leq 9$$

We need to use the largest possible value of y

so y = 9

x - y =

9 - -1 = 10

www.plexmaths.com