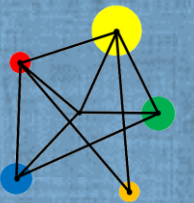


Angles in a Polygon



Find the angle sum of a polygon.

Calculate exterior and interior angles of a polygon.

Vocabulary

Polygon

A closed shape whose edges are all straight lines.

Interior Angle

An angle inside a shape, between two joined sides.

Vertically Opposite

The angles opposite each other when two lines cross. They are always equal.

Regular Polygon

A polygon with all angles and sides the same size.

Parallel

Lines that are always the same distance apart and never touching.

Exterior Angle

The angle between any side of a shape, and a line extended from the next side.

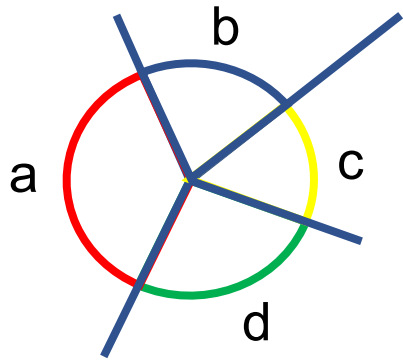
Vertex

Another name for a corner of a shape. The plural is vertices.

Equilateral

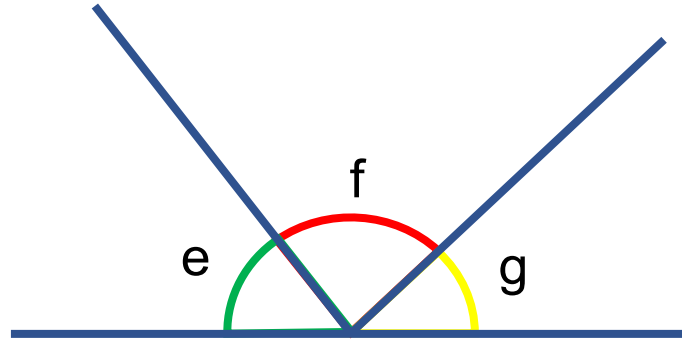
A name used for a regular triangle. Equilateral literally means 'equal sided'.

Review of Angle Facts



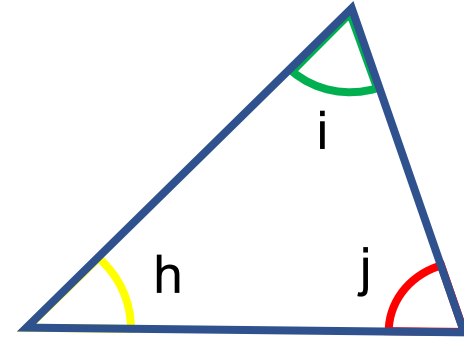
$$a + b + c + d = 360^\circ$$

Angles around a point add up to 360°



$$e + f + g = 180^\circ$$

Angles on a straight line add up to 180°

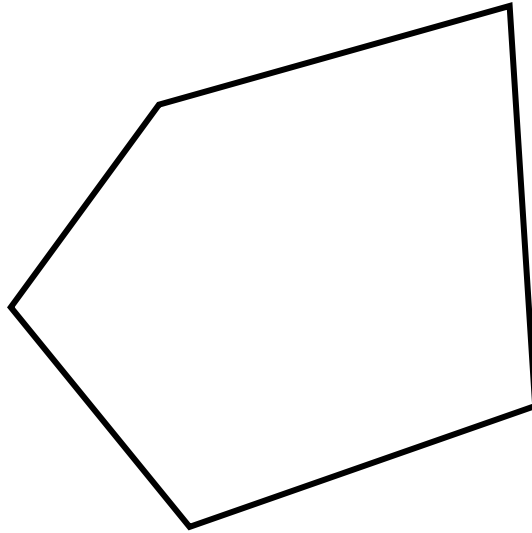


$$h + i + j = 180^\circ$$

Angles in a triangle add up to 180°

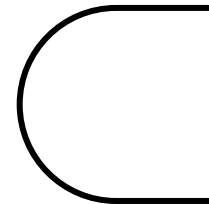
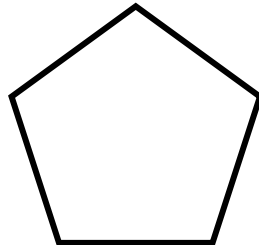
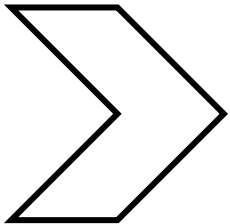
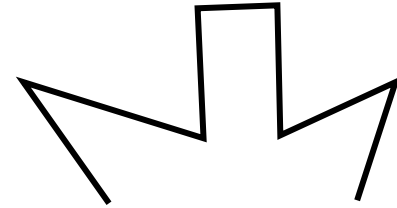
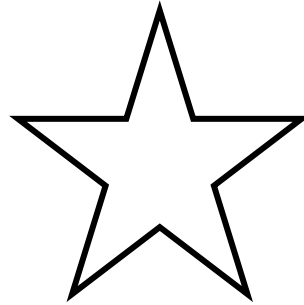
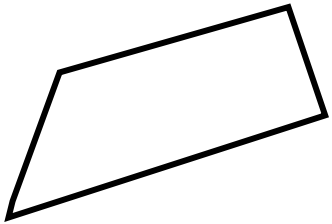
Key Fact

A **polygon** is a closed shape with straight edges.



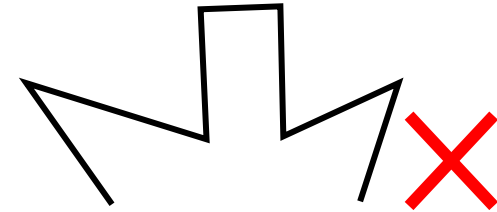
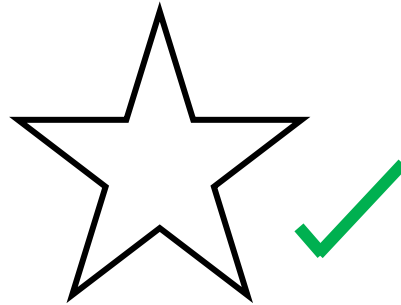
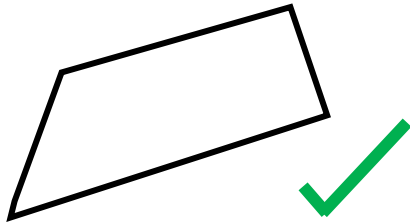
Question

Which of these shapes are polygons?.

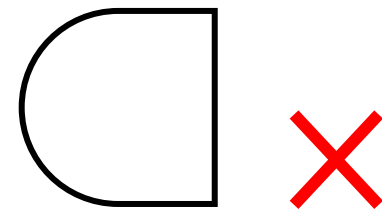
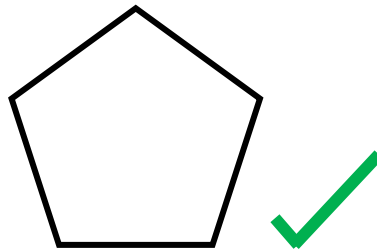
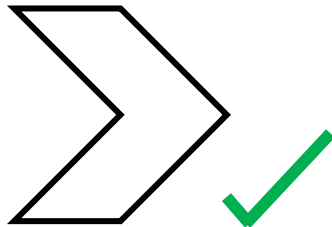


Question

Which of these shapes are polygons?



Not closed.

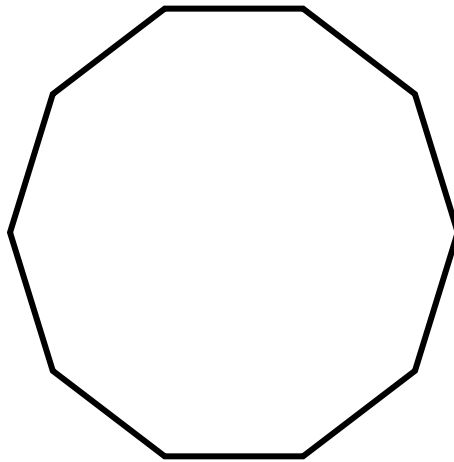


One edge is not straight.

Key Fact

A **regular polygon** is a polygon with all angles equal and all sides equal.

Here is a **regular decagon**.



10 sides

Names of Polygons

Sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon

Sides	Name
8	Octagon
9	Nonagon
10	Decagon
11	Hendecagon
12	Dodecagon

Match these regular polygons with their names.

Triangle

Hexagon

Quadrilateral

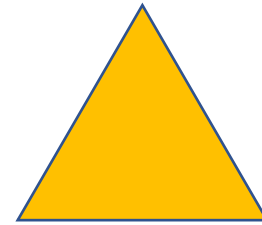
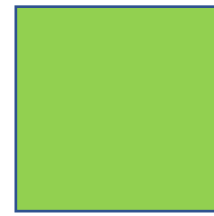
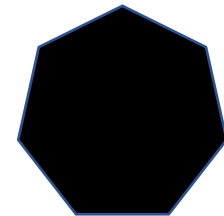
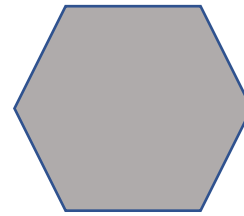
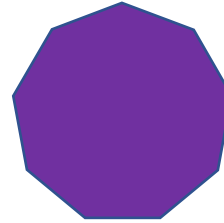
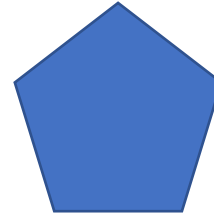
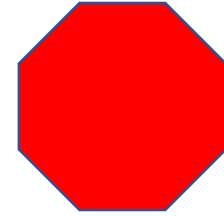
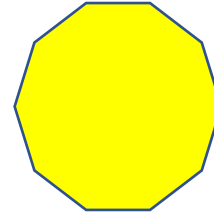
Octagon

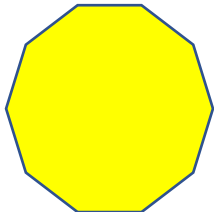
Pentagon

Nonagon

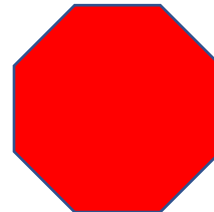
Heptagon

Decagon

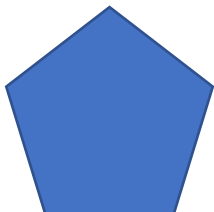




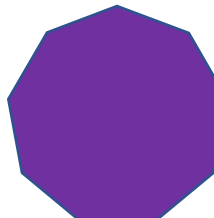
Decagon



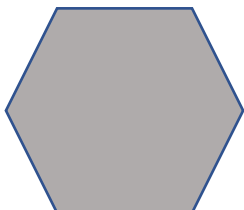
Octagon



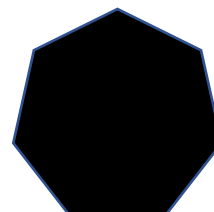
Pentagon



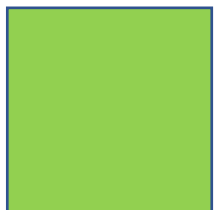
Nonagon



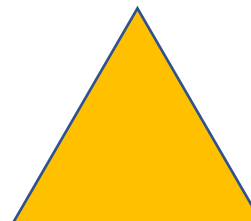
Hexagon



Heptagon



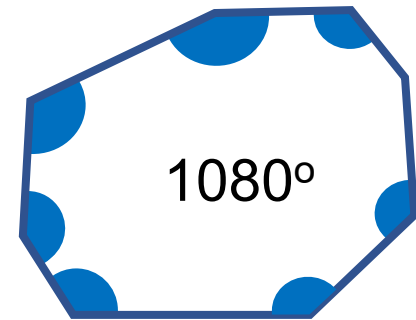
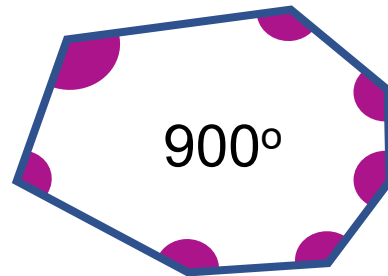
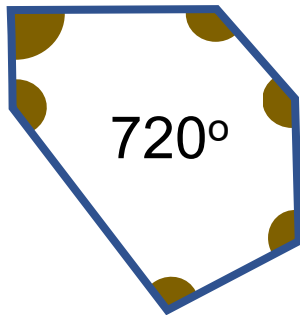
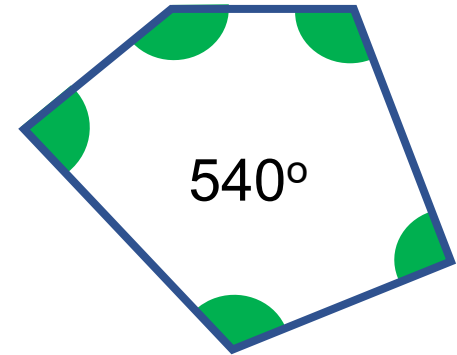
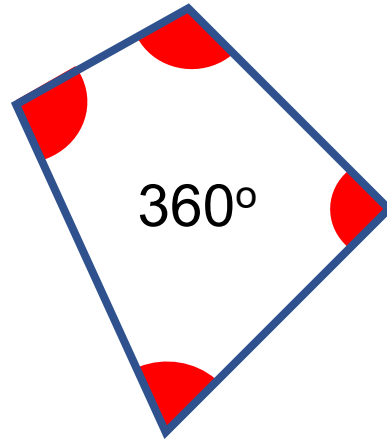
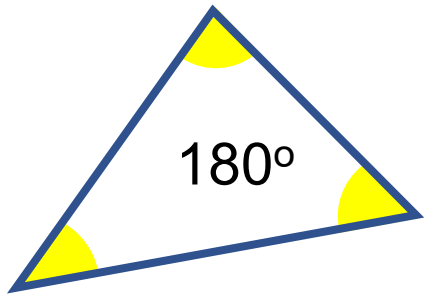
Quadrilateral



Triangle

Sums of angles in Polygons

The angle sum of a polygon depends on the number of sides.



Question

Can you find an expression for the sum of the angles in a polygon with 'n' sides?

No. of Sides	Sum of Angles
3	180°
4	360°
5	540°
6	720°
7	900°
8	1080°
9	1260°
n	?°

Solution

Can you find an expression for the sum of the angles in a polygon with 'n' sides?

No. of Sides	Sum of Angles
3	180°
4	360°
5	540°
6	720°
7	900°
8	1080°
9	1260°
n	$(n - 2) \times 180^\circ$

Key Facts

For a regular polygon with 'n' sides:

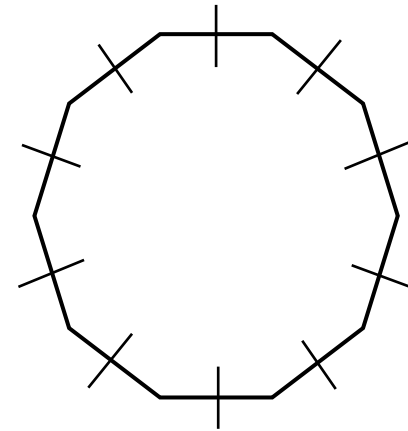
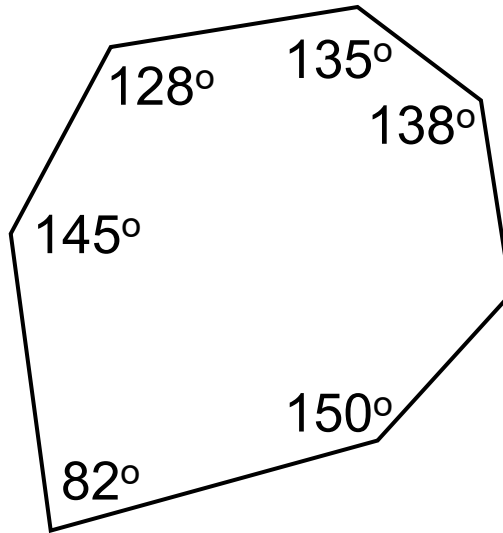
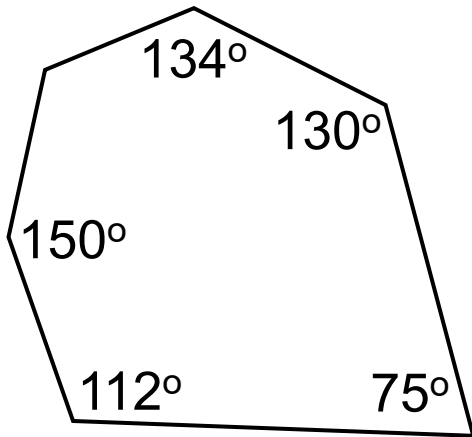
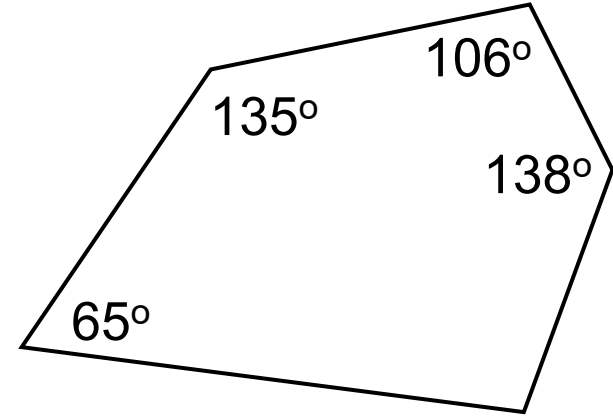
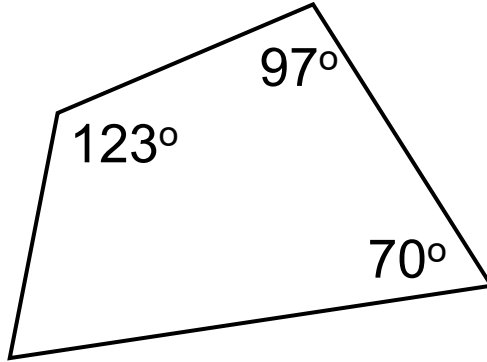
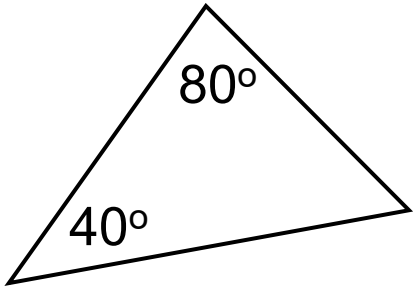
$$\text{Exterior angle} = \frac{360^\circ}{n}$$

$$\text{Interior angle} = 180 - \frac{360^\circ}{n}$$

$$\text{Sum of Angles} = (n - 2) \times 180$$

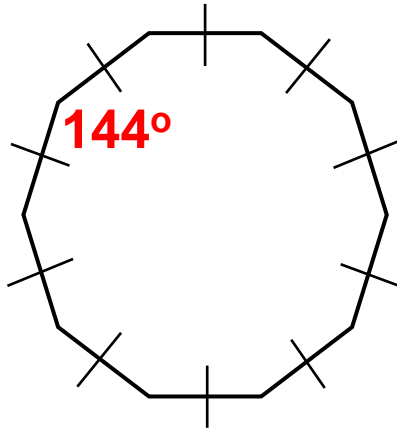
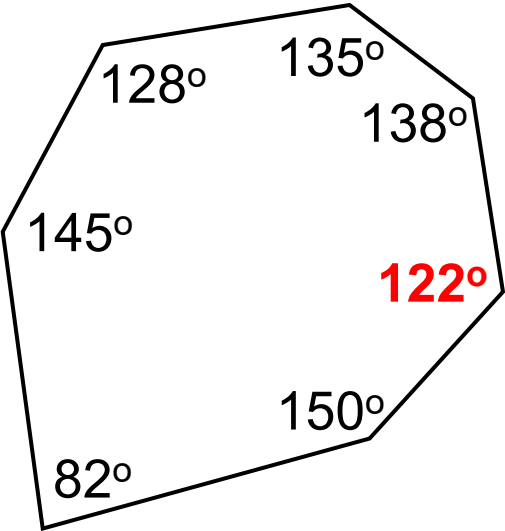
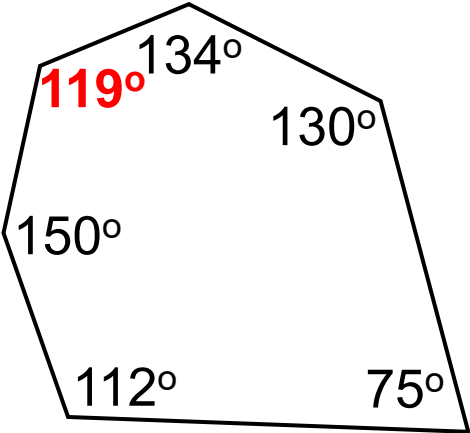
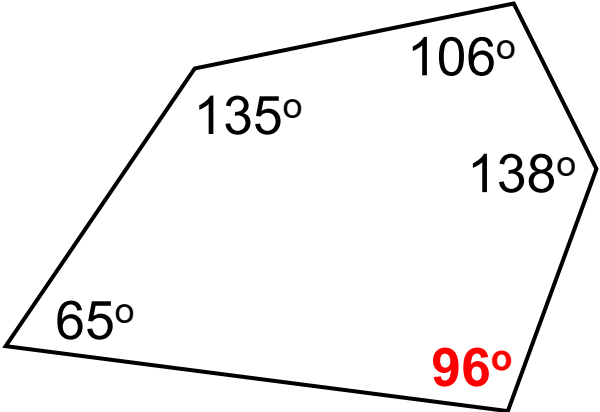
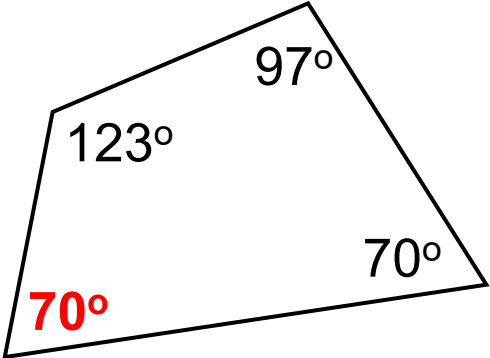
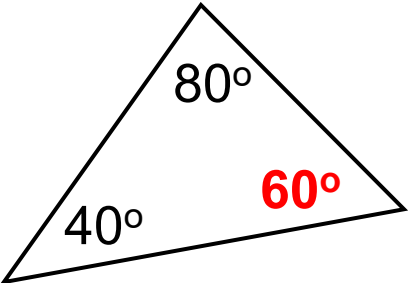
Exercise

Find the missing angles in each polygon.



Solutions

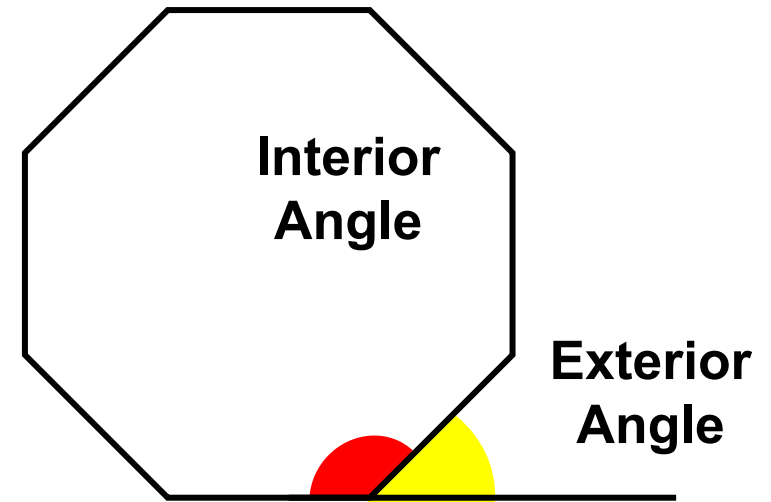
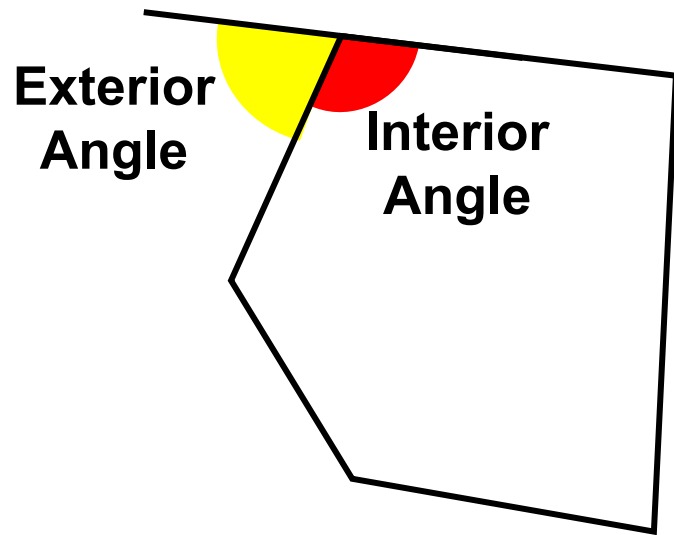
Find the missing angles in each polygon.



Definitions

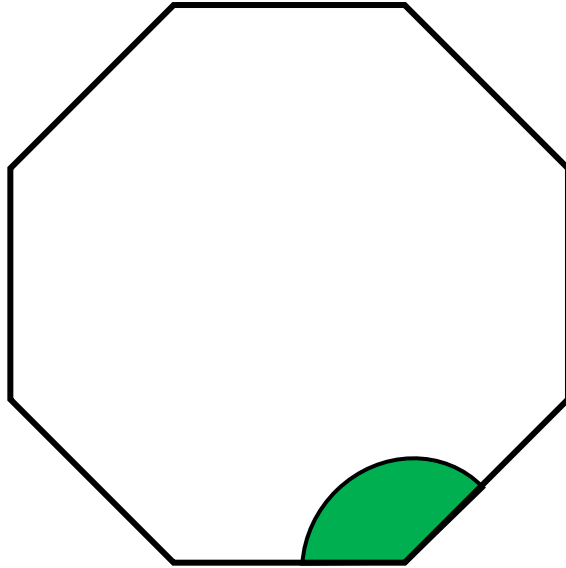
An interior angle is formed by the edges of a polygon. It is always inside the shape.

An exterior angle is formed by extending the edges of a polygon. It is always outside the shape.



Angles in a Regular Polygon

Here is a regular octagon.



Can you work out the size of the interior angle?

Method 1

There are eight exterior angles.

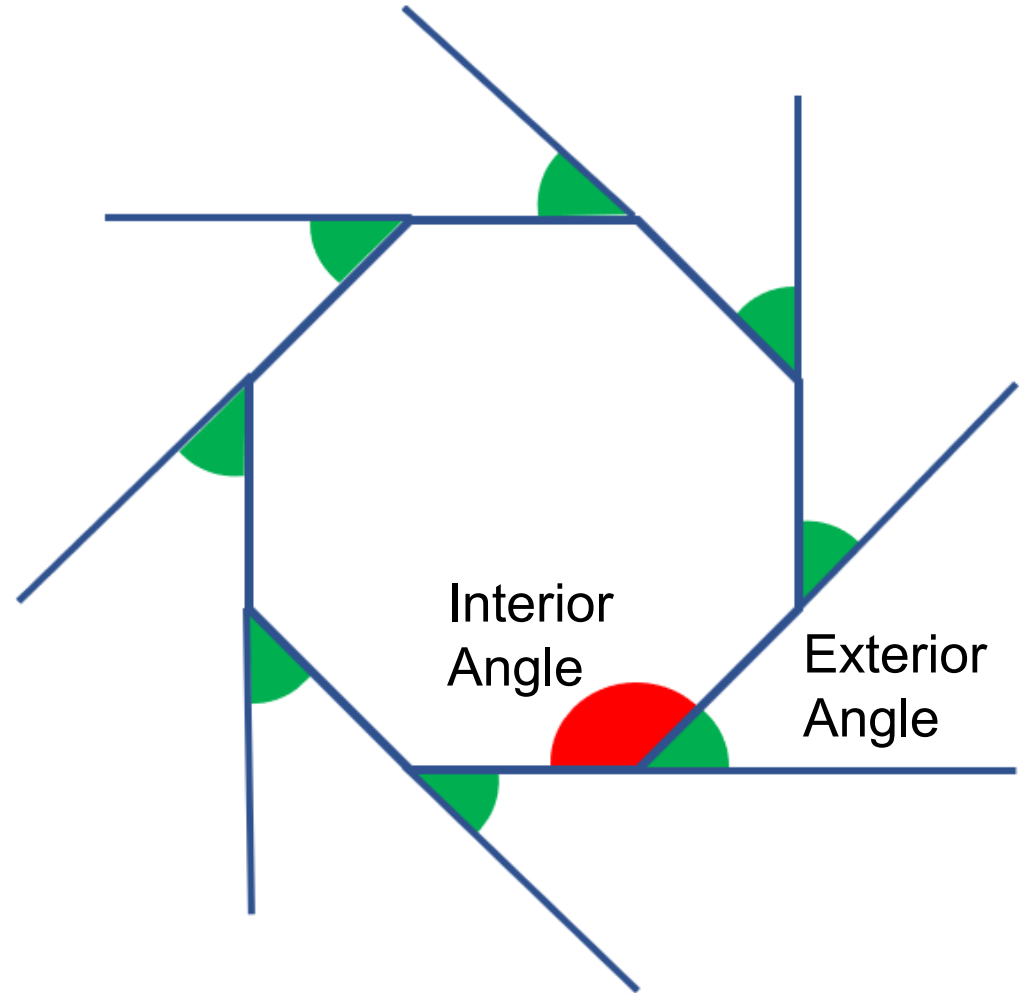
They are all the same size.

They add up to 360°

So, each one is $360 \div 8 = 45^\circ$.

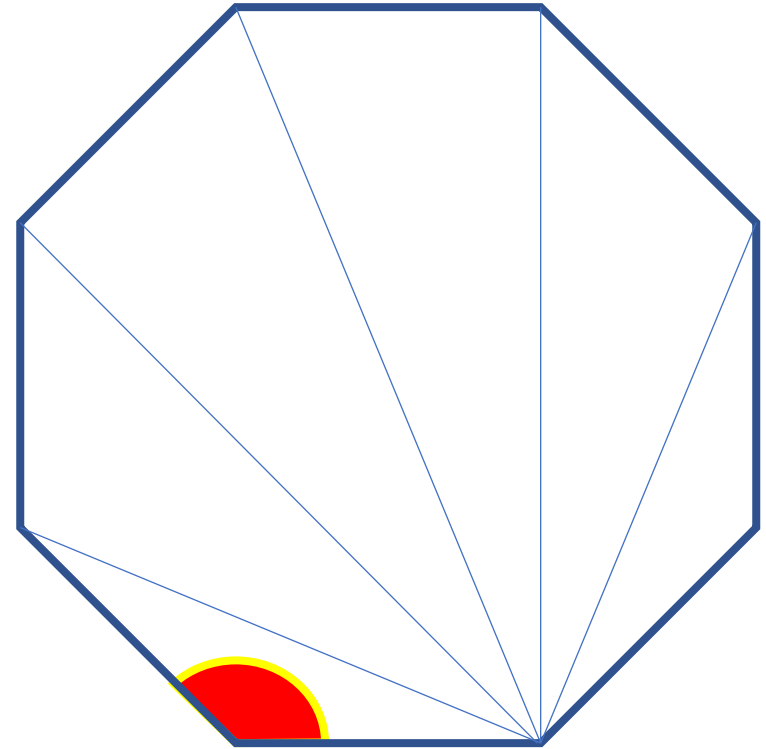
Angles on a straight line add up to 180°

Hence the interior angle must be 135°



Method 2

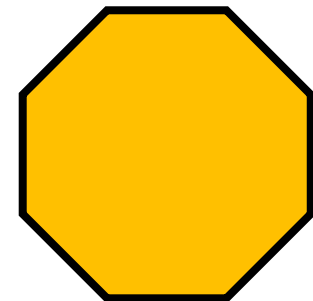
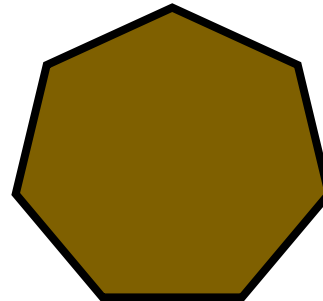
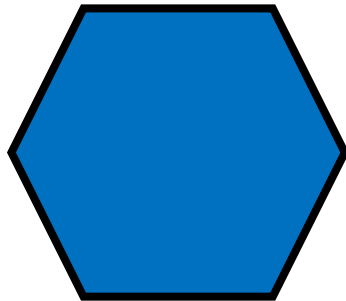
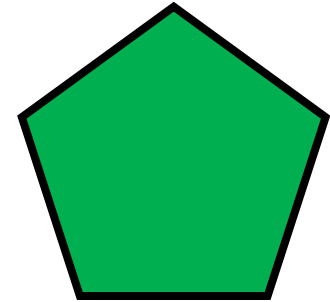
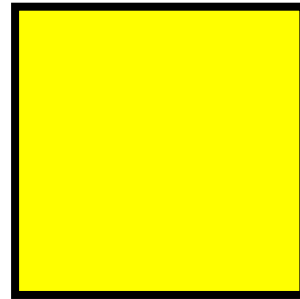
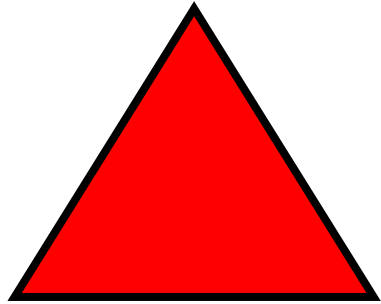
1. Pick any vertex
2. Draw lines to all the other vertices.
3. This splits the octagon into 6 triangles.
4. $6 \times 180^\circ = 1080^\circ$
5. So, each interior angle is $1080^\circ \div 8 = 135^\circ$



Exercise

Find the size of the interior angles in these regular polygons.

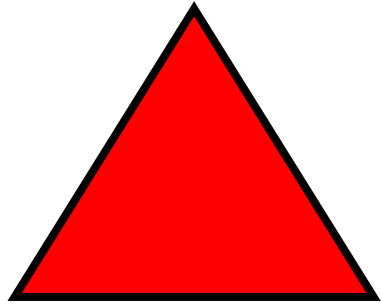
Once you have found the size of an interior angle, accurately draw the regular polygon.



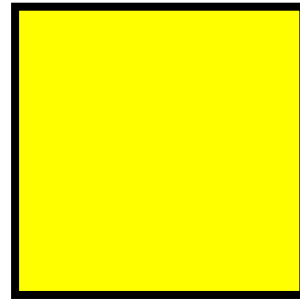
Solutions

Find the size of the interior angles in these regular polygons.

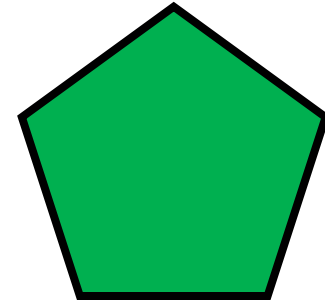
Once you have found the size of an interior angle, accurately draw the regular polygon.



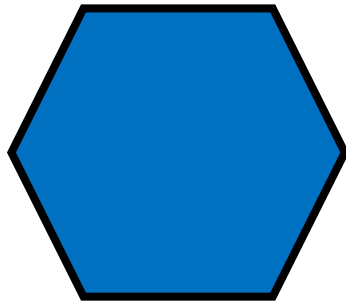
60°



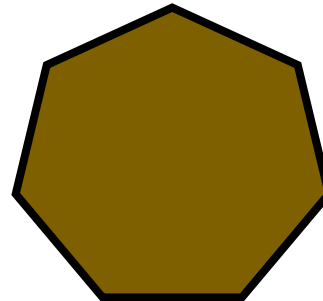
90°



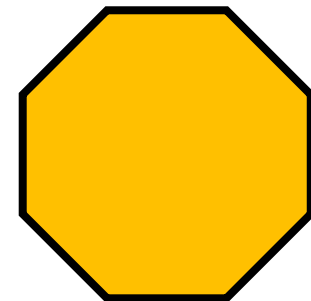
108°



120°



128.6°



135°

What is the more common name for a regular triangle?

What is the more common name for a regular quadrilateral?

What is the sum of the angles in a quadrilateral?

Which has the larger exterior angle – a pentagon or a hexagon?

Which has the larger interior angle – an octagon or a decagon?

What is the size of each exterior angle of a regular octagon?

What is the sum of the interior angles of a pentagon?

The sum of interior angles in a regular polygon is 720° . How many sides does it have and what is its name?

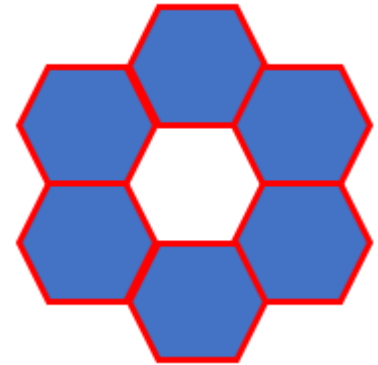
What is the size of each interior angle of a regular nonagon?

What is the sum of the interior angles of a dodecagon (12 sides).

Each interior angle in a regular polygon is 144° . How many sides does it have and what is its name?

Challenge

Regular hexagons can be joined together to make a ring,



Can you make this pattern with other regular polygons?

What is the more common name for a regular triangle?

Equilateral

What is the more common name for a regular quadrilateral?

Square

What is the sum of the angles in a quadrilateral?

360°

Which has the larger exterior angle – a pentagon or a hexagon?

Pentagon°

Which has the larger interior angle – an octagon or a decagon?

Decagon

What is the size of each exterior angle of a regular octagon?

45°

What is the sum of the interior angles of a pentagon?

540°

The sum of interior angles in a regular polygon is 720°. How many sides does it have and what is its name?

6, Regular Hexagon

What is the size of each interior angle of a regular nonagon?

140°

What is the sum of the interior angles of a dodecagon (12 sides).

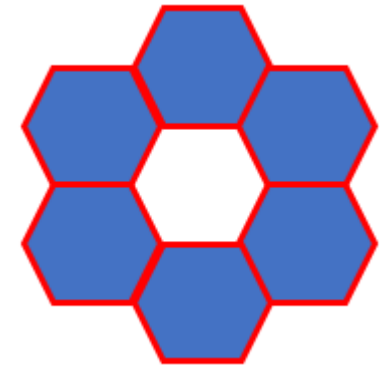
1800°

Each interior angle in a regular polygon is 144°. How many sides does it have and what is its name?

10, Decagon

Challenge

Regular hexagons can be joined together to make a ring,



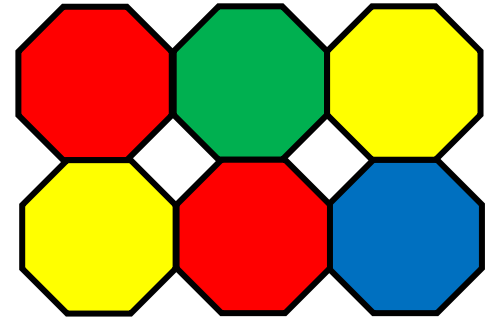
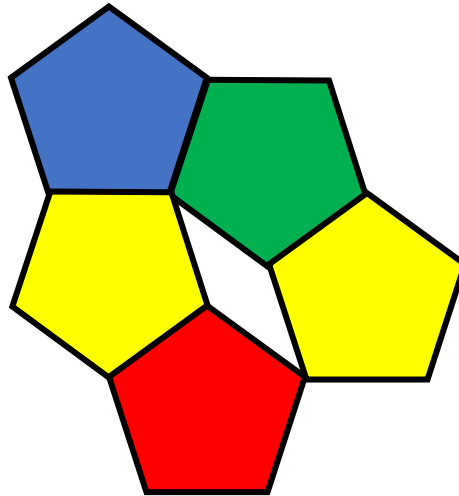
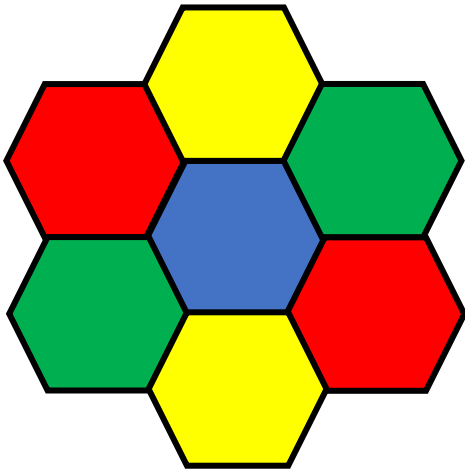
Can you make this pattern with other regular polygons?

Investigation

It is possible to tile a floor with regular hexagons and leave no gaps.

It is not possible to tile a floor with regular pentagons or octagons. Why?

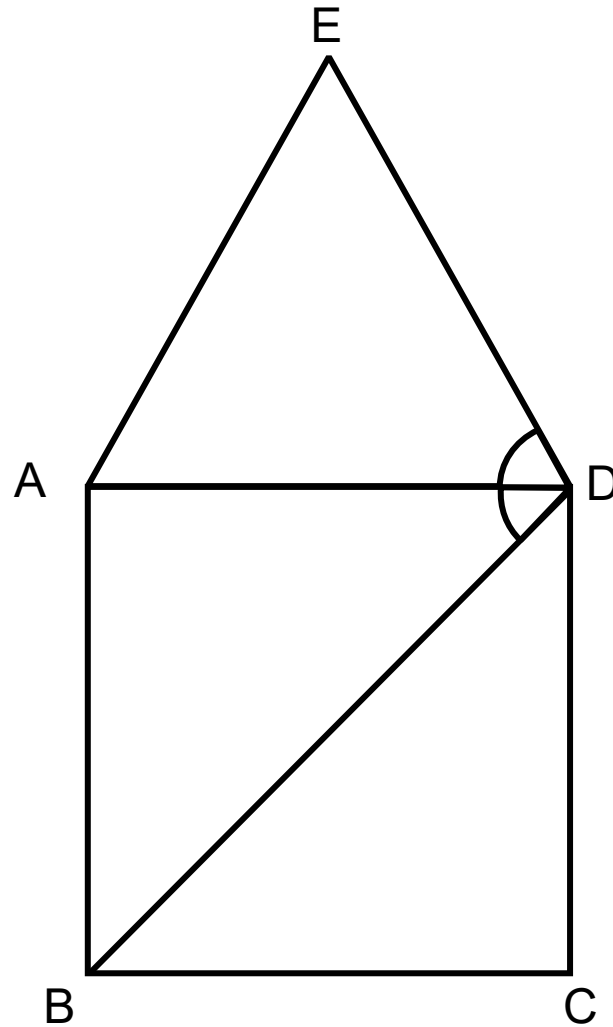
Which regular polygons can be used to tile a floor and why?



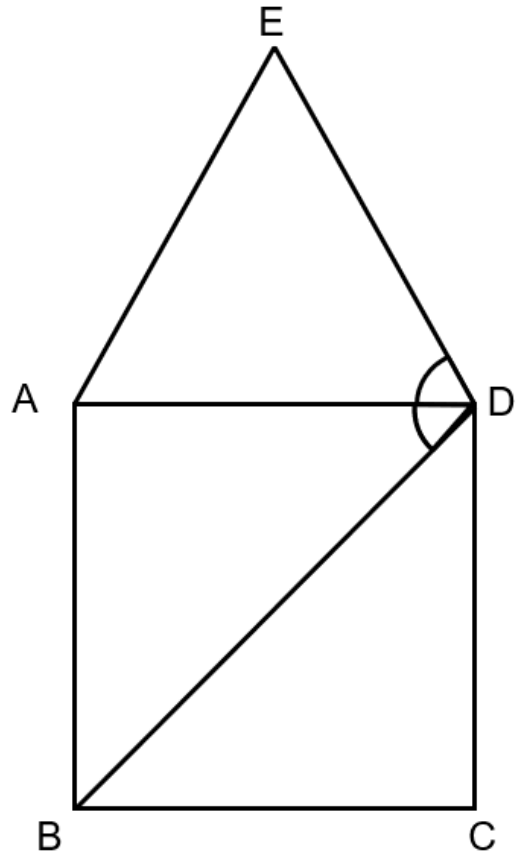
Exam Style Question

The diagram shows a square $ABCD$ and an equilateral triangle ADE .

Find the size of angle BDE .



Solution



Find the size of angle BDE

Angle ADC = 90° (angle in a square)

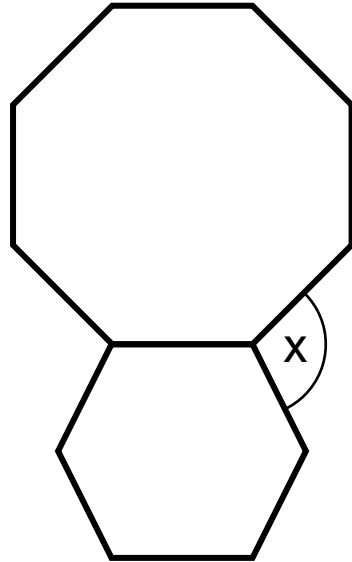
Angle ADB = 45° (half of angle ADC)

Angle ADE = 60° (angle in an equilateral triangle)

Angle BDE = 45° + 60° = 105°

Exam Style Question

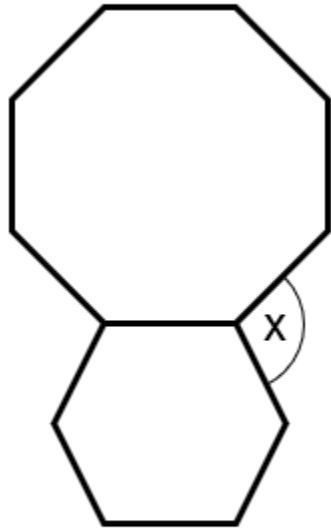
A regular octagon and a regular hexagon are joined along one side.



Work out the size of the angle marked x .
You must show all of your working.

Solution

A regular octagon and a regular hexagon are joined along one side.



Work out the size of the angle marked x .
You must show all of your working.

Angle in regular octagon = 135°

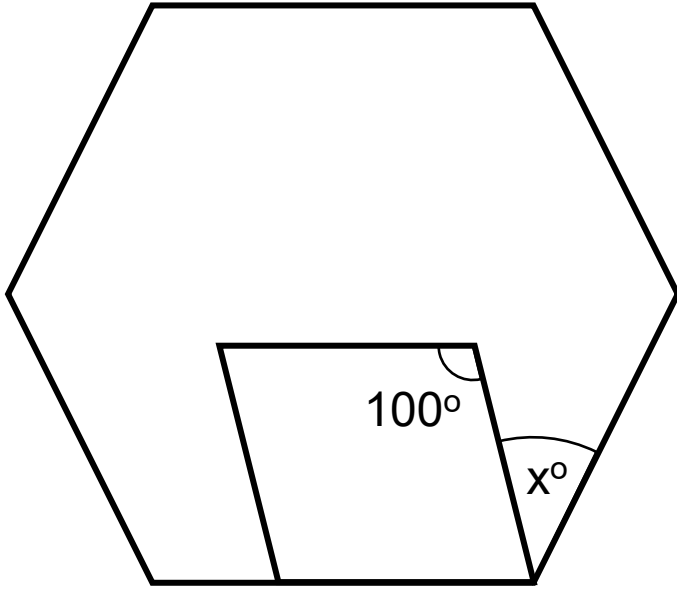
Angle in regular hexagon = 120°

Angle $x = 360^\circ - (135^\circ + 120^\circ)$

$= 105^\circ$

Exam Style Question

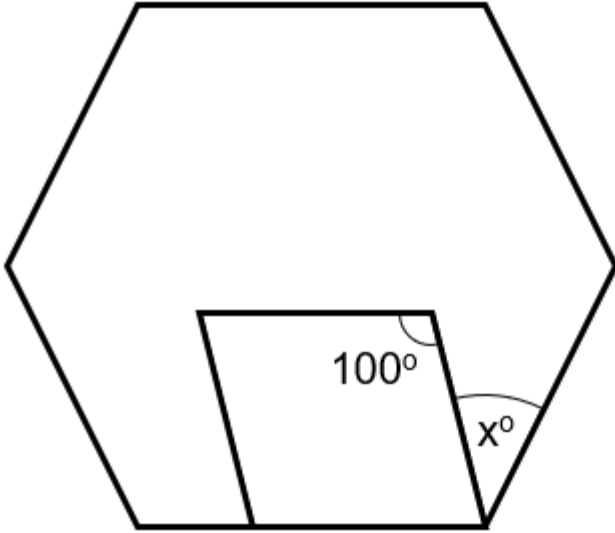
The diagram shows a regular hexagon and a parallelogram.



Work out the size of the angle marked x .
You must show all of your working.

Solution

The diagram shows a regular hexagon and a parallelogram.



Work out the size of the angle marked x .
You must show all of your working.

$$\text{Angle in regular hexagon} = 120^\circ$$

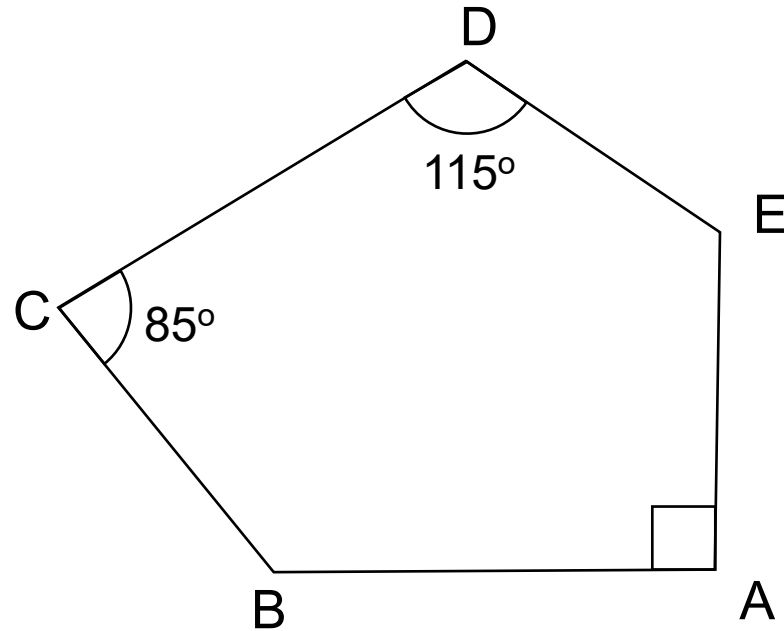
$$\text{Angle in parallelogram} = 80^\circ$$

$$\text{Angle } x = 120^\circ - 80^\circ$$

$$= \underline{40^\circ}$$

Exam Style Question

ABCDE is a pentagon.

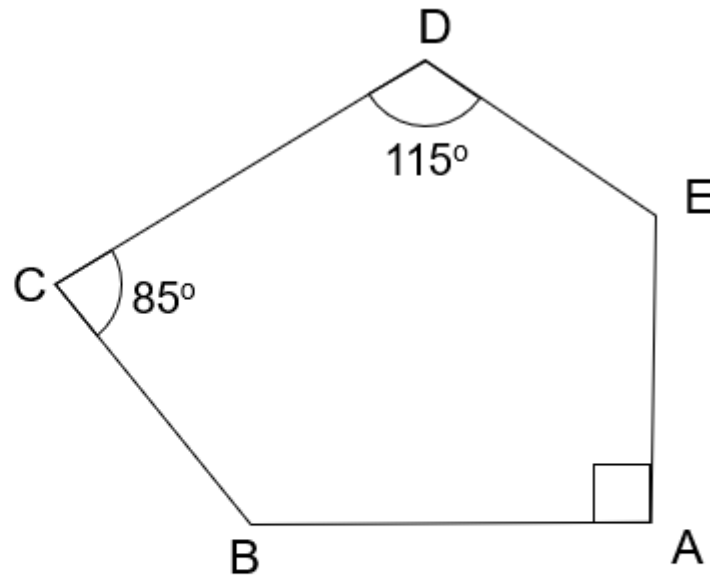


Angle ABC = Angle AED.

Work out the size of angle ABC.
You must show all of your working.

Solution

ABCDE is a pentagon.



Angle ABC = Angle AED.

Work out the size of angle ABC.

You must show all of your working.

*Angles in pentagon
have a sum of 540°*

$$540^\circ - 85^\circ - 115^\circ - 90^\circ = 250^\circ$$

$$250^\circ \div 2 = 125^\circ$$

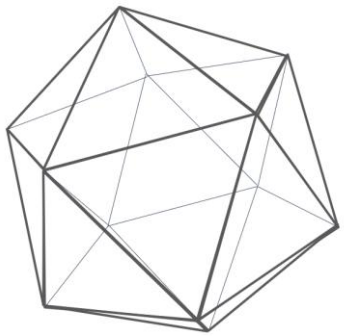
Angle ABC = 125°

Where next?

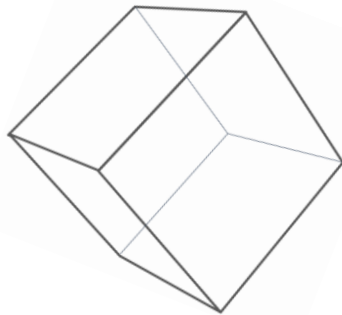
3 Dimensional Geometry

The study of angles in polygons allows us to consider three-dimensional shapes. The solids below are the **Platonic solids**. These are three-dimensional solid shapes which have regular polygons as each of their faces.

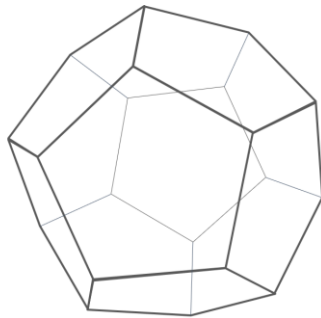
The Platonic solids were investigated over 2000 years ago by the Greek philosopher Plato. He discovered and proved that there are only five of them. You might like to investigate the Platonic solids and see if you can understand why there are only five of them.



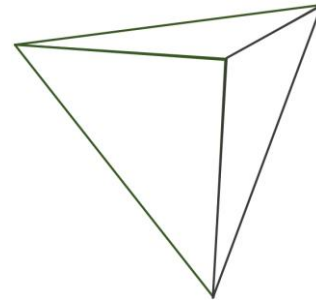
Icosahedron



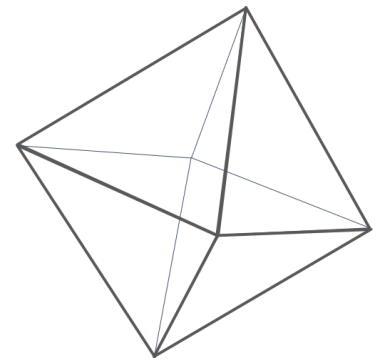
Cube



Dodecahedron



Tetrahedron



Octahedron

Historical links

The word polygon consists of two parts – ‘poly’ and ‘gon’. ‘Poly’ derives from the Greek adjective πολύς (polús) 'many' and ‘gon’ derives from γωνία (gōnía) which means 'angle'. Some people think that γόνυ (gónu) 'knee' may be the origin of gon as an angle is reminiscent of a knee.

The names used for polygons are derived from the Greek words for numbers.

Number	Greek name	Polygon name
3	Tria	Triangle
4	Tessera (quad comes from latin)	Quadrilateral
5	Pendi	Pentagon
6	Exi	Hexagon
7	Efta	Heptagon
8	Ocho	Octagon
9	Ennea	Nonagon