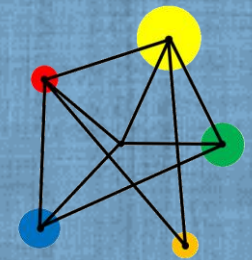


Linear Sequences



Understand the meaning of linear and non-linear

Extend linear sequences

Construct linear sequences from an n 'th term expression

Find an n 'th term expression for a sequence

Vocabulary

Term

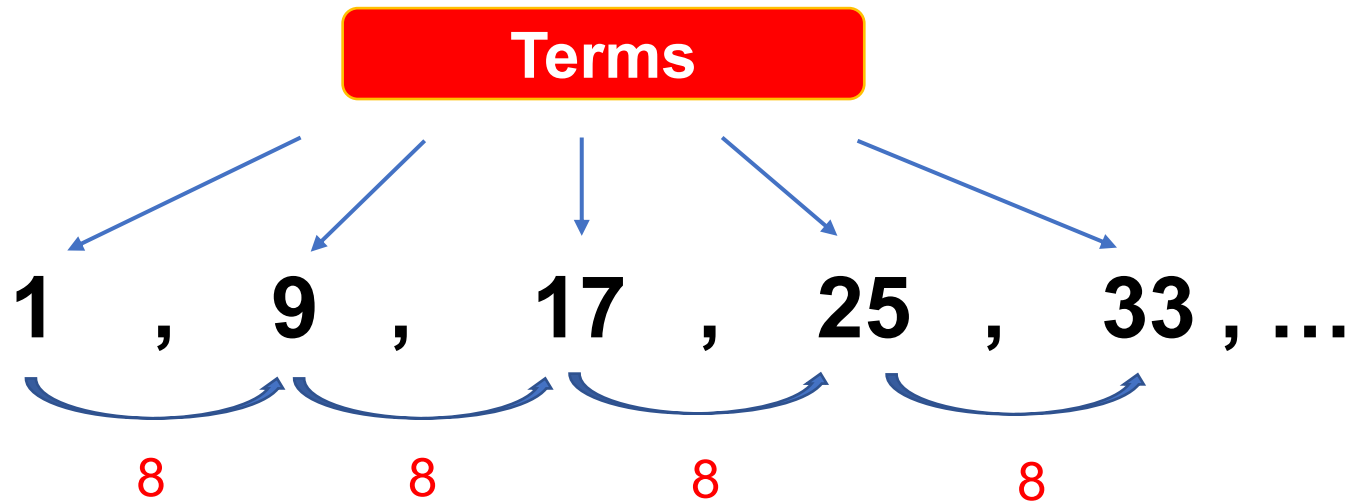
A number in a sequence.

Linear

The terms in a linear sequence increase or decrease by the same amount.

Key Facts

A **linear** sequence is made up of 'terms'.



In a linear sequence the difference between terms is always the same.

Example

1 , 9 , 17 , 25 , 33 , _____ , _____

The term-to-term rule for this sequence is 'add 8'.

Use the term-to-term rule to find the next two terms.

Solution

1 , 9 , 17 , 25 , 33 , 41 , 49

The term to term rule for this sequence is 'add 8'.

Use the term to term rule to find the next two terms.

Exercise

Write down the next two terms of each linear sequence.

- a) 1 , 3 , 5 , 7 ...
- b) 1 , 4 , 7 , 10 ...
- c) 6 , 11 , 16 , 21 ...
- d) 18 , 27 , 36 , 45 ...
- e) -8 , -5 , -2 , 1 ...
- f) 14 , 11 , 8 , 5 ...

Write down the next two terms of each linear sequence.

- a) -13 , -9 , -5 , -1 ...
- b) 42 , 31 , 20 , 9 ...
- c) 4 , 5.9 , 7.8 , 9.7 ...
- d) $\frac{1}{2}$, $1\frac{1}{4}$, 2 , $2\frac{3}{4}$...
- e) $\frac{1}{10}$, $\frac{3}{10}$, $\frac{1}{2}$, $\frac{7}{10}$, ...

These are non-linear sequences. Can you write down the next two terms of each one?

- a) 1 , 4 , 9 , 16 ...
- b) 1 , 3 , 6 , 10 ...
- c) 1 , 1 , 2 , 3 , 5 ...
- d) 1 , 8 , 27 , 64 ...

Solutions

Write down the next two terms of each linear sequence.

a) 1, 3, 5, 7, **9, 11**

b) 1, 4, 7, 10, **13, 16**

c) 6, 11, 16, 21, **26, 31**

d) 18, 27, 36, 45, **54, 63**

e) -8, -5, -2, 1, **4, 7**

f) 14, 11, 8, 5, **2, -1**

Write down the next two terms of each linear sequence.

a) -13, -9, -5, -1, **3, 7**

b) 42, 31, 20, 9, **-2, -13**

c) 4, 5.9, 7.8, 9.7, **11.6, 13.5**

d) $\frac{1}{2}$, $1\frac{1}{4}$, 2, $2\frac{3}{4}$, **$3\frac{1}{2}$, $4\frac{1}{4}$**

e) $\frac{1}{10}$, $\frac{3}{10}$, $\frac{1}{2}$, $\frac{7}{10}$, **$\frac{9}{10}$, $\frac{11}{10}$**

These are non-linear sequences. Can you write down the next two terms of each one?

a) 1, 4, 9, 16, **25, 36**

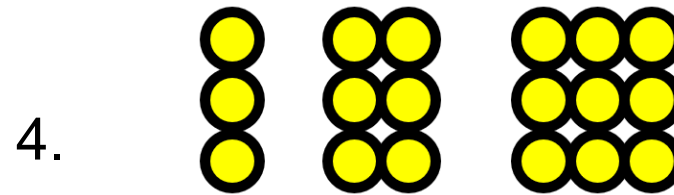
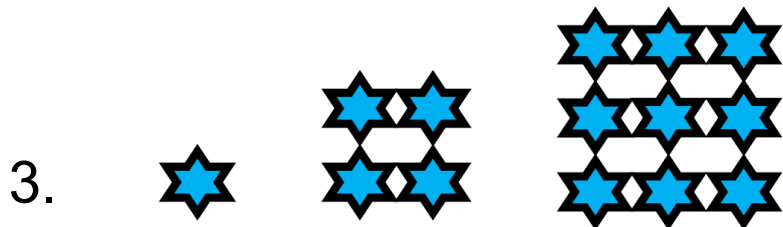
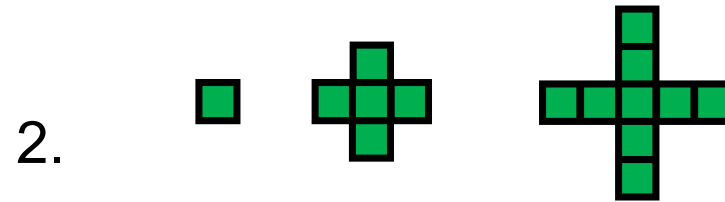
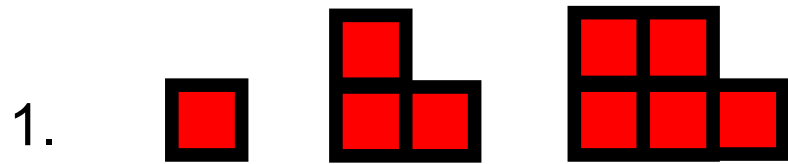
b) 1, 3, 6, 10, **15, 21**

c) 1, 1, 2, 3, 5, **8, 13**

d) 1, 8, 27, 64, **125, 216**

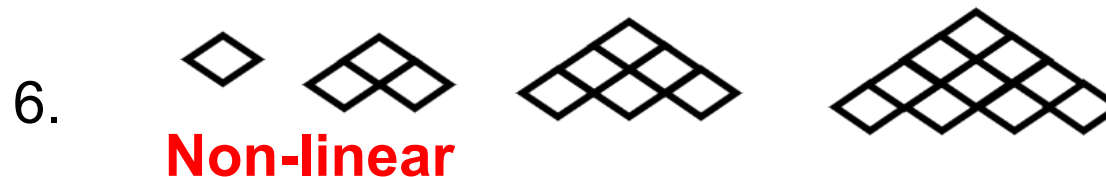
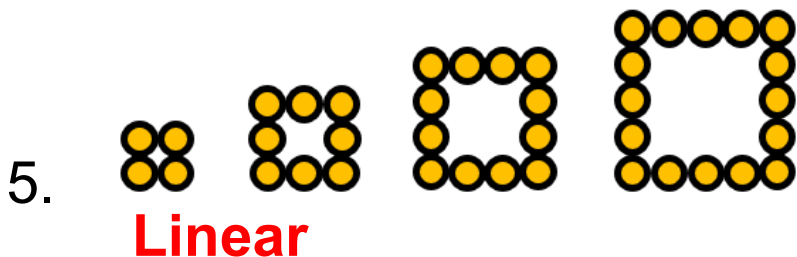
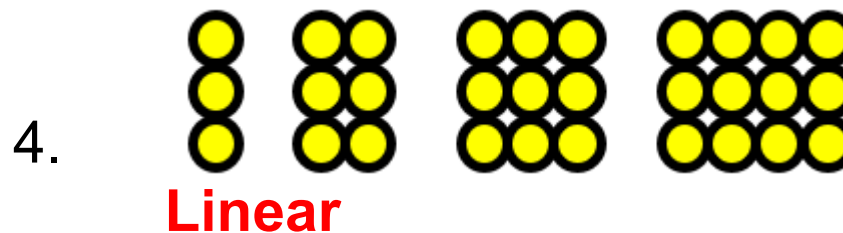
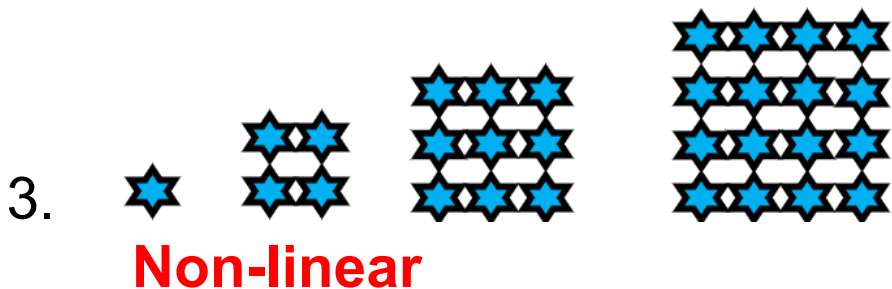
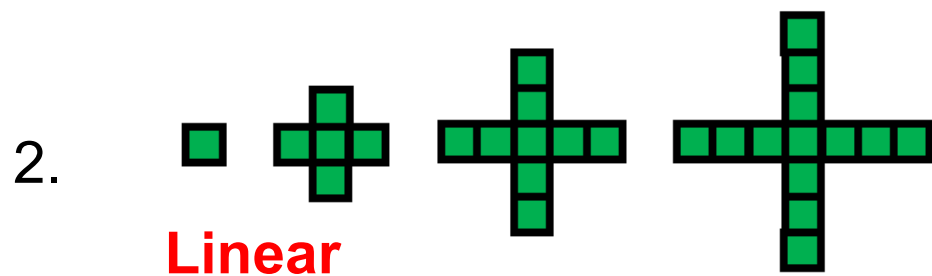
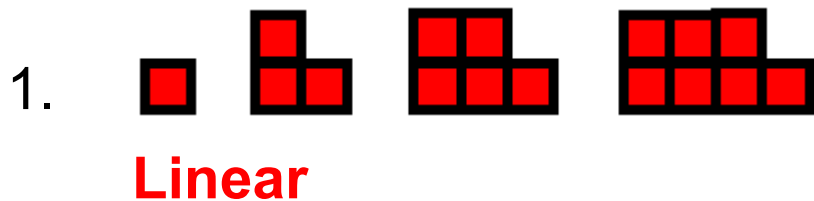
Picture Sequences

Copy each diagram and draw the next pattern in each one.
Decide whether each sequence is linear or not.



Solutions

Copy each diagram and draw the next pattern in each one.
Decide whether each sequence is linear or not.



Graphs

We can plot the terms of a linear sequence on a graph.

Position in sequence	1	2	3	4	5
Term	3	5	7	9	11

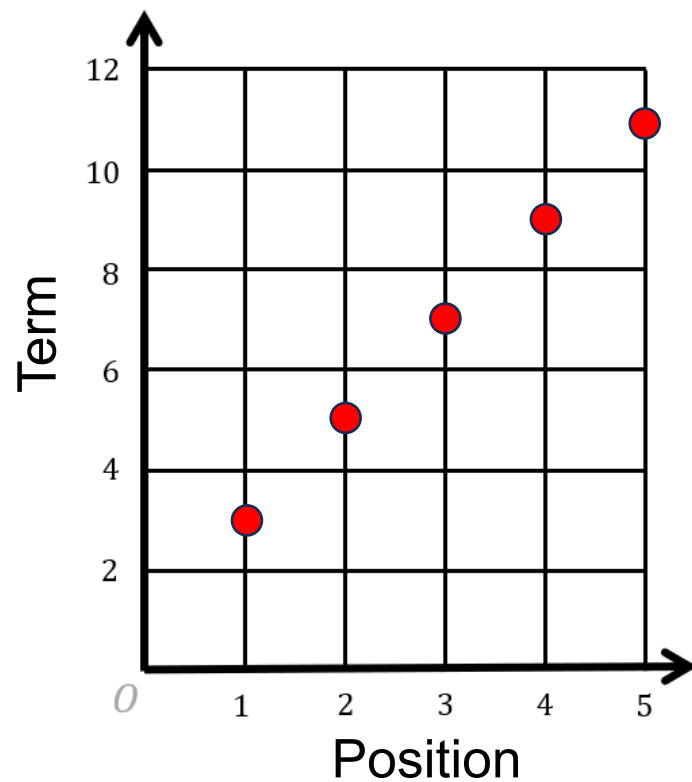


Diagram Sequences

Plot the terms of each sequence on a graph.

Are any of the sequences non-linear?

a)

Position	1	2	3	4	5
Term	1	3	5	7	9

b)

Position	1	2	3	4	5
Term	2	3	4	5	6

c)

Position	1	2	3	4	5
Term	5	10	15	20	25

d)

Position	1	2	3	4	5
Term	1	4	8	13	19

e)

Position	1	2	3	4	5
Term	10	8	6	4	2

f)

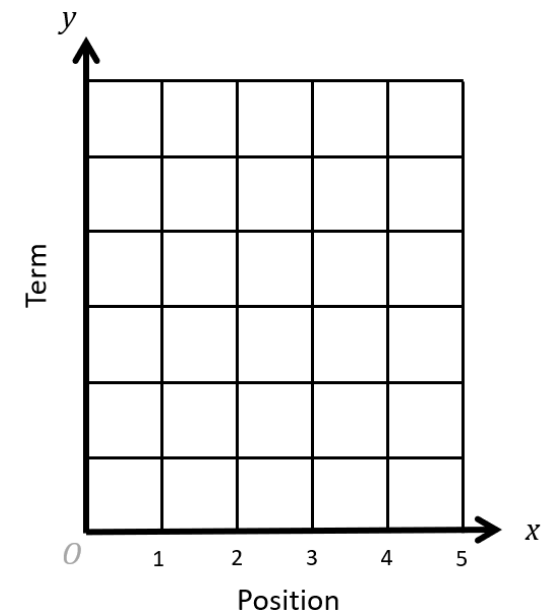
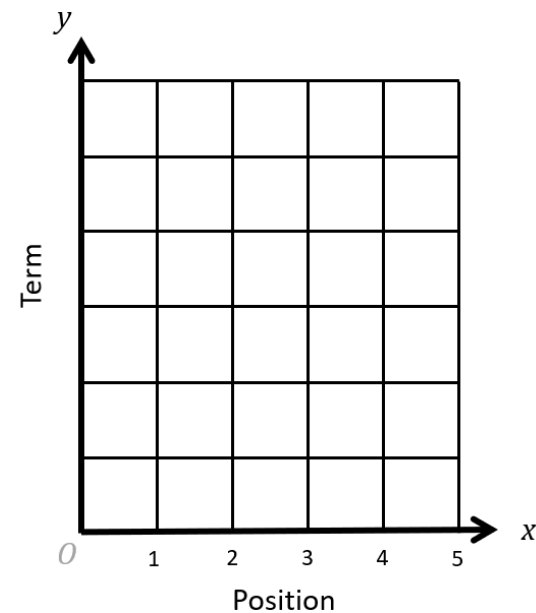
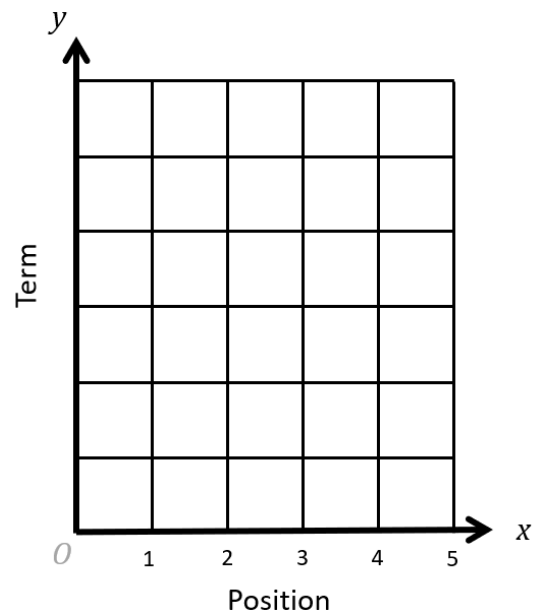
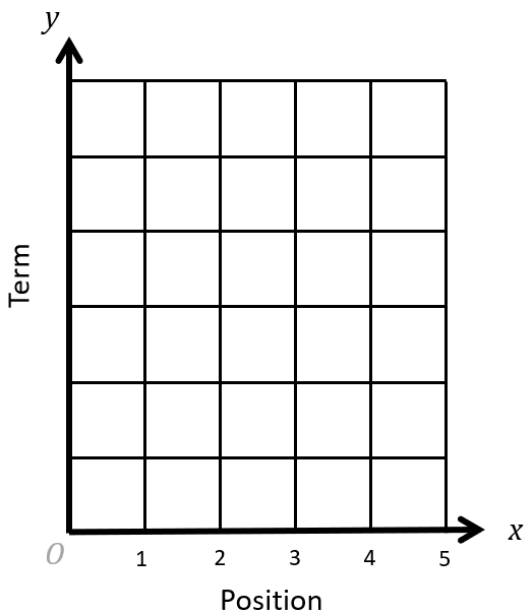
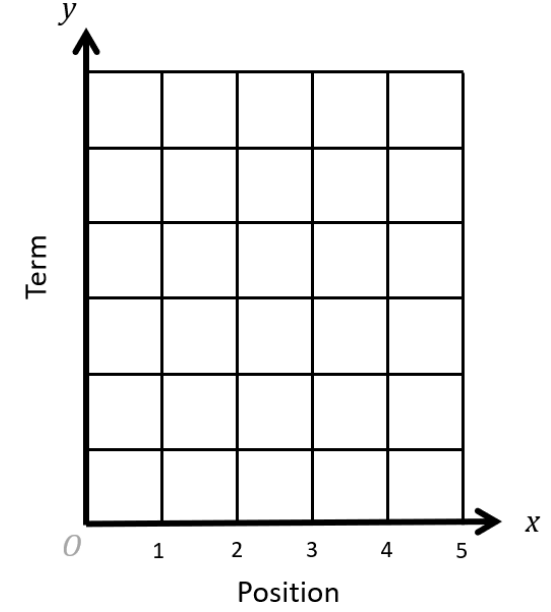
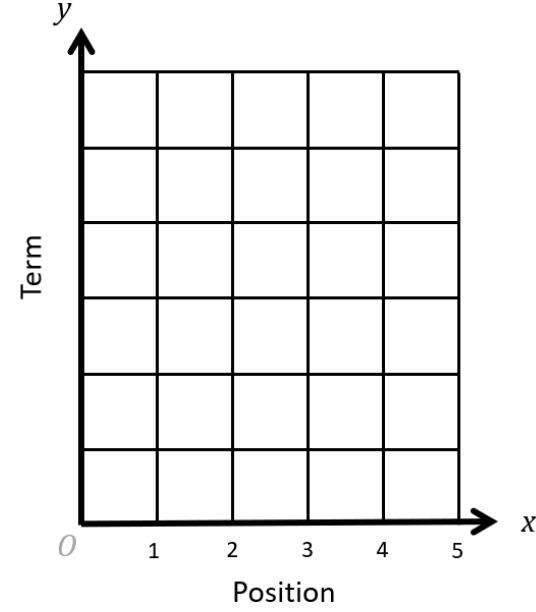
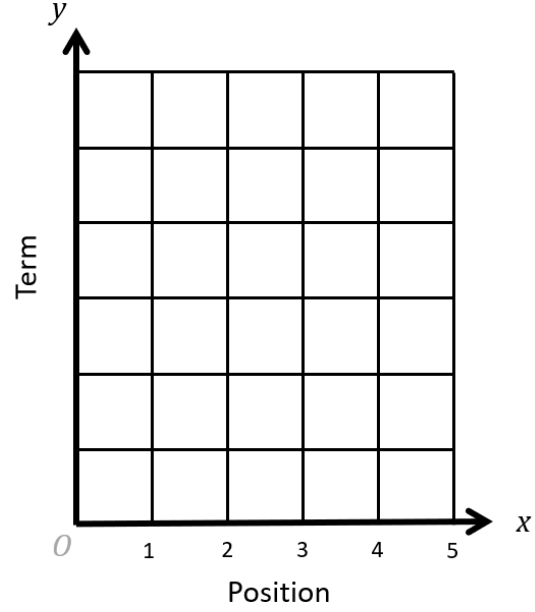
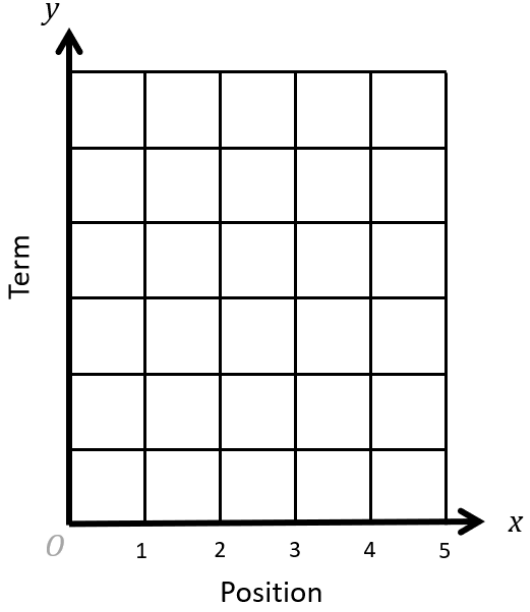
Position	1	2	3	4	5
Term	1	5	9	13	17

g)

Position	1	2	3	4	5
Term	3	8	13	18	23

h)

Position	1	2	3	4	5
Term	6	6	6	6	6

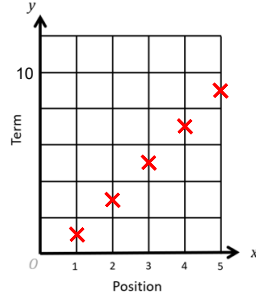


Solutions

a)

Position	1	2	3	4	5
Term	1	3	5	7	9

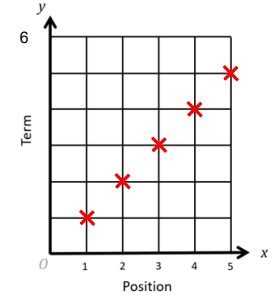
Linear



b)

Position	1	2	3	4	5
Term	2	3	4	5	6

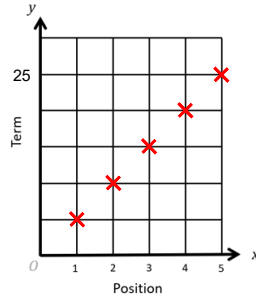
Linear



c)

Position	1	2	3	4	5
Term	5	10	15	20	25

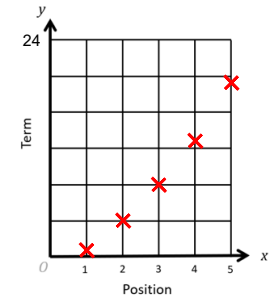
Linear



d)

Position	1	2	3	4	5
Term	1	4	8	13	19

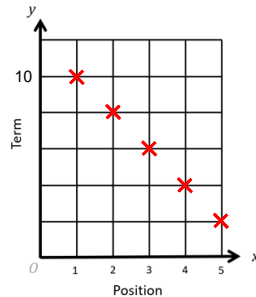
Non-linear



e)

Position	1	2	3	4	5
Term	10	8	6	4	2

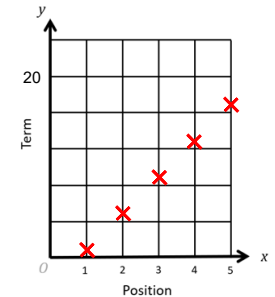
Linear



f)

Position	1	2	3	4	5
Term	1	5	9	13	17

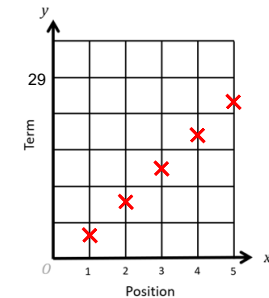
Linear



g)

Position	1	2	3	4	5
Term	3	8	13	18	23

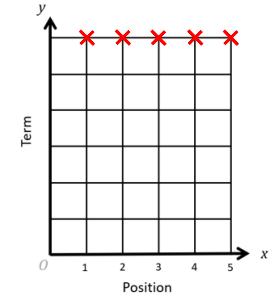
Linear



h)

Position	1	2	3	4	5
Term	6	6	6	6	6

Linear



Exercise

Find the missing numbers in each linear sequence

- a) 2 , 5 , 8 , 11 , ? , ?
- b) 5 , 10 , 15 , 20 , ? , ?
- c) 3 , 5 , ? , ? , 11
- d) ? , ? , 13 , 19 , 25 , 31
- e) 20 , 17 , ? , ? , 8
- f) 10 , ? , 18 , ? , 26

Find the missing numbers in each linear sequence

- a) 13 , 10 , 7 , 4 , ? , ?
- b) 1 , ? , ? , 25 , 33 , 41
- c) 8 , ? , 16 , ? , 24
- d) 7 , ? , ? , ? , 27
- e) 13 , ? , ? , ? , -3

Find the missing numbers in each linear sequence

- a) 1 , ? , ? , ? , 11
- b) 0.3 , ? , ? , ? , 5.5
- c) 0 , ? , ? , ? , 1

Solutions

Find the missing numbers in each linear sequence

- a) 2 , 5 , 8 , 11 , **14** , **17**
- b) 5 , 10 , 15 , 20 , **25** , **30**
- c) 3 , 5 , **7** , **9** , 11
- d) **1** , **7** , 13 , 19 , 25 , 31
- e) 20 , 17 , **14** , **11** , 8
- f). 10 , **14** , 18 , **22** , 26

Find the missing numbers in each linear sequence

- a) 13 , 10 , 7 , 4 , **1** , **- 2**
- b) 1 , **9** , **17** , 25 , 33 , 41
- c) 8 , **12** , 16 , **20** , 24
- d) 7 , **12** , **17** , **22** , 27
- e) 13 , **9** , **5** , **1** , - 3

Find the missing numbers in each linear sequence

- a) 1 , **3.5** , **6** , **8.5** , 11
- b) 0.3 , **1.6** , **2.9** , **4.2** , 5.5
- c) 0 , $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1

Key Fact

Sequences can be defined by an expression.

For example, a sequence is defined by the expression $2n + 1$

n	$2n + 1$
1	3
2	5
3	7
4	9
5	11
100	201

'n' tells us which term in the sequence we are looking at.

So, the sequence defined by $2n + 1$ begins : 3 , 5 , 7 , 9 , 11 ...

You try

Write down the first five terms of the sequences defined by the expressions below:

a) $2n + 4$

b) $12 - 3n$

Write down the hundredth terms of each sequence:

Solutions

Write down the first five terms of the sequences defined by the expressions below:

a) $2n + 4$ **6, 8, 10, 12, 14**

$12 - 3n$ **9, 6, 3, 0, -3**

Write down the hundredth terms of each sequence:

a) **$200 + 4 = 204$**

b) **$12 - 300 = -288$**

Exercise

Write down the first four terms and the hundredth term of each sequence.

1. $n + 5$

2. $2n$

3. $10 + n$

4. $8 - n$

5. $5n + 2$

6. $n - 3$

Write down the first four terms and the hundredth term of each sequence.

1. $10n$

2. $1 - n$

3. $\frac{n}{2}$

4. n^2

5. n^3

Which of these sequences are non-linear?

Write down the first four terms and hundredth term of each non-linear sequence.

1. $n(n + 1)$

2. $\frac{12}{n}$

3. 2^n

4. $\left(1 + \frac{1}{n}\right)^n$

Solutions

Write down the first four terms and the hundredth term of each sequence.

1. $n + 5$ **6, 7, 8, 9 ... 105**

2. $2n$ **2, 4, 6, 8 ... 200**

3. $10 + n$ **11, 12, 13, 14 ... 110**

4. $8 - n$ **7, 6, 5, 4 ... -92**

5. $5n + 2$ **7, 12, 17, 22 ... 502**

6. $n - 3$ **-2, -1, 0, 1 ... 97**

Write down the first four terms and the hundredth term of each sequence.

1. $10n$
10, 20, 30, 40 ... 1000

2. $1 - n$
0, -1, -2, -3 ... -99

3. $\frac{n}{2}$
0.5, 1, 1.5, 2 ... 50

4. n^2
1, 4, 9, 16 ... 10 000

5. n^3
1, 8, 27, 64 ... 1 000 000

Which of these sequences are non-linear? **4 and 5**

Write down the first four terms and hundredth term of each non-linear sequence.

1. $n(n + 1)$ **2, 6, 12, 20 ... 10 100**

2. $\frac{12}{n}$ **12, 6, 4, 3 ... 0.12**

3. 2^n **2, 4, 8, 16 ... 1.27 x 10³⁰**

4. $\left(1 + \frac{1}{n}\right)^n$
2, 2.25, 2.3, 2.44 ... 2.70481382942

Question

What is the 100th term of this sequence?

1 , 3 , 5 , 7 , 9 , 11 , ...

Solution

We could write out 100 terms of the sequence:

1 , 3 , 5 , 7 , 9 , 11 , 13 , 15 , 17 , 19 , 21 , 23 , 25 , 27 , 29 , 31 , 33 , 35 ,
37 , 39 , 41 , 43 , 45 , 47 , 49 , 51 , 53 , 55 , 57 , 59 , 61 , 63 , 65 , 67 , 69
, 71 , 73 , 75 , 77 , 79 , 81 , 83 , 85 , 87 , 89 , 91 , 93 , 95 , 97 , 99 , 101 ,
103 , 105 , 107 , 109 , 111 , 113 , 115 , 117 , 119 , 121 , 123 , 125 , 127 ,
129 , 131 , 133 , 135 , 137 , 139 , 141 , 143 , 145 , 147 , 149 , 151 , 153 ,
155 , 157 , 159 , 161 , 163 , 165 , 167 , 169 , 171 , 173 , 175 , 177 , 179 ,
181 , 183 , 185 , 187 , 189 , 191 , 193 , 195 , 197 , **199**

The hundredth term is **199**.

Writing down 100 terms will take a long time.

If we know an expression for the sequence:

$$2n - 1$$

Then we can substitute $n = 100$:

$$2 \times 100 - 1 = 199$$

This is much quicker but how can we find an expression?

Finding an expression

Discuss how you might find an expression for this linear sequence.

2 , 7 , 12 , 17 , 22 , ...

$$2, 7, 12, 17, 22, \dots$$

A sequence of numbers: 2, 7, 12, 17, 22, followed by an ellipsis. Red arcs connect each pair of adjacent terms. Below each arc is the number 5, indicating that the difference between consecutive terms is 5.

Each term is 5 greater than the previous term

That is the same as the sequence $5n$

$$5, 10, 15, 20, 25, \dots$$

A sequence of numbers: 5, 10, 15, 20, 25, followed by an ellipsis. Red arcs connect each pair of adjacent terms. Below each arc is the number 5, indicating that the difference between consecutive terms is 5.

The expression for our sequence is $5n - 3$

You try:

Find an expression for these linear sequences.

a) 5 , 7 , 9 , 11 , 13 , ...

b) 23 , 18 , 13 , 8 , 3 , ...

Solution

Find an expression for these linear sequences.

a) 5 , 7 , 9 , 11 , 13 , ... **$2n + 3$**

b) 23 , 18 , 13 , 8 , 3 , ... **$-5n + 28$ or $28 - 5n$**

Find the n'th term formula & the 100th term for each linear sequence.

1. 3, 5, 7, 9.....

2. 8, 12, 16, 20.....

3. 3, 7, 11, 13.....

4. 0, 5, 10, 15.....

5. 6, 12, 18, 24.....

6. 14, 12, 10, 8.....

Find the n'th term formula and the 100th term for each linear sequence.

1. -7, -4, -1, 3.....

2. 1, 2, 3, 4.....

3. 7, 7, 7, 7.....

4. 2, 2.2, 2.4, 2.6.....

5. $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, ...

6. 1, 0, -1, -2.....

Find an n'th term formula for these non-linear sequences

1. 1, 4, 9, 16, 25.....

2. 1, 3, 6, 10, 15.....

3. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$

Challenge

A linear sequence has 1st term 12 and 20th term 50. Find the n'th term expression and the hundredth term.

Find the n'th term formula & the 100th term for each linear sequence.

1. 3, 5, 7, 9.....
 $2n + 1$, 201
2. 8, 12, 16, 20.....
 $4n + 4$, 404
3. 3, 7, 11, 15.....
 $4n - 1$, 399
4. 0, 5, 10, 15.....
 $5n - 5$, 495
5. 6, 12, 18, 24.....
 $6n$, 600
6. 14, 12, 10, 8.....
 $16 - 2n$, -184

Find the n'th term formula and the 100th term for each linear sequence.

1. -7, -4, -1, 3.....
 $10 - 3n$, -290
2. 1, 2, 3, 4.....
 n , 100
3. 7, 7, 7, 7.....
 $0n + 7$, 7
4. 2, 2.2, 2.4, 2.6.....
 $0.2n + 1.8$, 21.8
5. $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, ...
 $\frac{1}{4}n + \frac{1}{4}$, $25\frac{1}{4}$
6. 1, 0, -1, -2.....
 $2 - n$, -98

Find an n'th term formula for these non-linear sequences

1. 1, 4, 9, 16, 25.....
 n^2
2. 1, 3, 6, 10, 15.....
 $\frac{n(n+1)}{2}$
3. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$
 $\frac{2}{n+1}$

Challenge

A linear sequence has 1st term 12 and 20th term 50. Find the n'th term expression and the hundredth term.

$2n + 10$, 210

Find the n'th term expression & the 100th term for each linear sequence.

a) 5, 8, 11, 14.....

b) 6, 8, 10, 12.....

c) 1, 5, 9, 13.....

d) 7, 12, 17, 22,

e) 3, 6, 9, 12.....

f) -1, 3, 7, 11.....

g) 8, 17, 26, 35.....

h) 10, 7, 4, 1.....

i) 1.5, 2, 2.5, 3.....

j) 12, 8, 4, 0.....

k) 1.6, 2.4, 3.2, 4.....

l) 7, 9, 11, 13.....

Find the n'th term expression & the 100th term for each linear sequence.

a) 5 , 8 , 11 , 14.....

$3n + 2$

b) 6 , 8 , 10 , 12.....

$2n + 4$

c) 1 , 5 , 9 , 13

$4n - 3$

d) 7 , 12 , 17 , 22 ,

$5n + 2$

e) 3 , 6 , 9 , 12.....

$3n$

f) -1 , 3 , 7 , 11.....

$4n - 5$

g) 8 , 17 , 26 , 35.....

$9n - 1$

h) 10 , 7 , 4 , 1.....

$13 - 3n$

i) 1.5 , 2 , 2.5 , 3.....

$0.5n + 1$

j) 12 , 8 , 4 , 0.....

$16 - 4n$

k) 1.6 , 2.4 , 3.2 , 4.....

$0.8n + 0.8$

l) 7 , 9 , 11 , 13.....

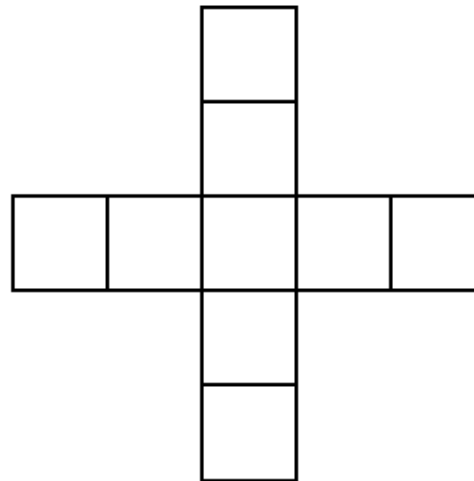
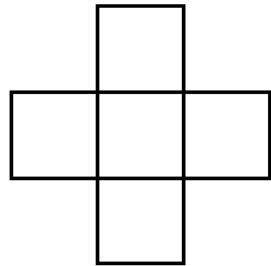
$2n + 5$

Extension

Can you continue the pattern below?

What would the 100th pattern look like?

How many squares would it contain?

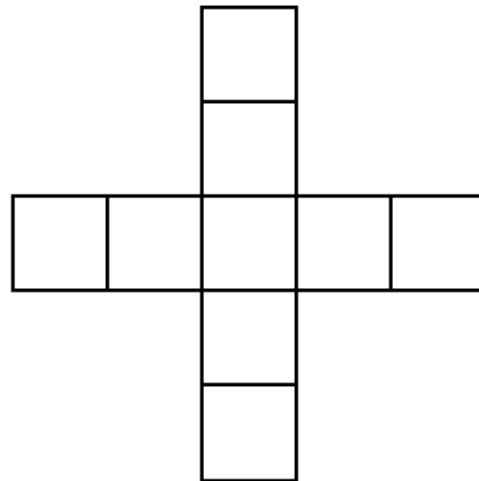
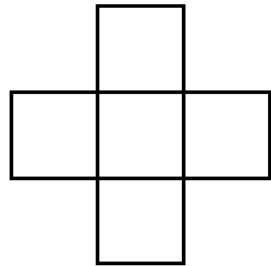


Solution

Can you continue the pattern below?

What would the 100th pattern look like?

How many squares would it contain?



The 100th pattern would contain **401** squares.

Exam Style Question

Here are the first six terms of a linear sequence.

4 10 16 22 28 34

a) Find an expression, in terms of n , for the n th term of this sequence.

The n th term of a different sequence is $4n^2$.

Larissa says that the 4th term of this sequence is 256

b) Is Larissa right?

Show how you get your answer.

Solution

Here are the first six terms of a linear sequence.

4 10 16 22 28 34

a) Find an expression, in terms of n , for the n th term of this sequence.

$$6n - 2$$

The n th term of a different sequence is $4n^2$.

Larissa says that the 4th term of this sequence is 256

b) Is Larissa right?

Show how you get your answer.

Larissa is not correct.

$$4 \times 4^2 = 64, \text{ not } 256.$$